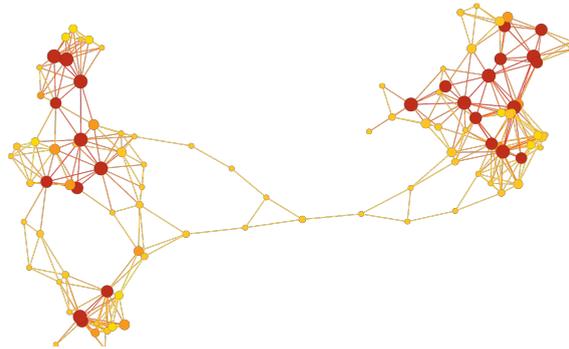


# Topology and Machine Learning

A Global Map of your Data

Anthony Bak

## AYASDI



# Outline

## Part I

- ▶ Problems with Big Data

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- ▶ Review

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Caveats: I am only talking about the strain of TDA done by Ayasdi

Goals

TDA Review

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The shape (segmentations, groupings) represent verified hypothesis. You have to decide if they are interesting.

# Math World

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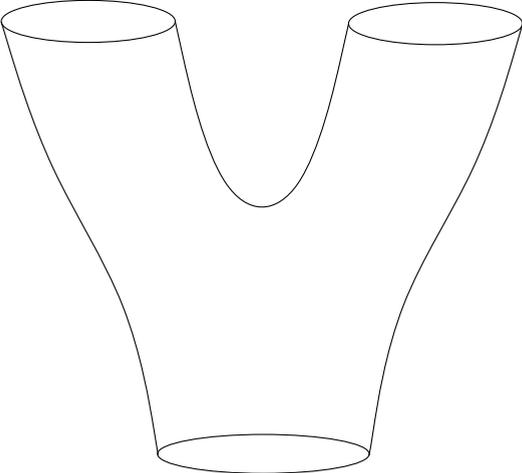
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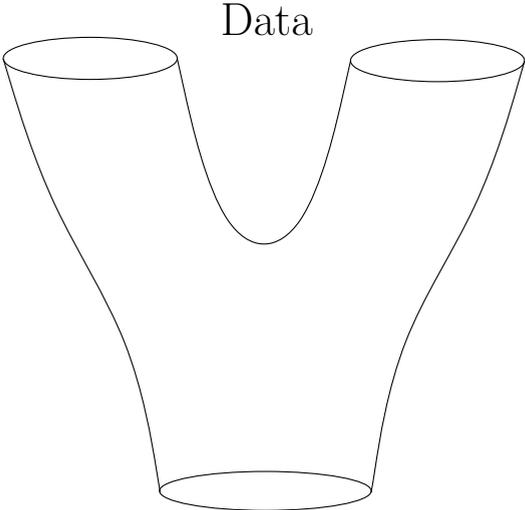
⇒ Even more importantly, data in the real world is **never** like this.

Math World

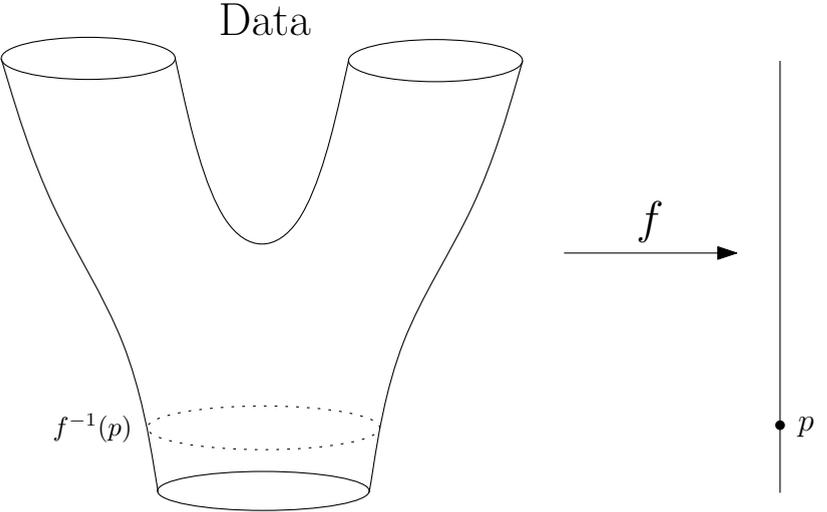
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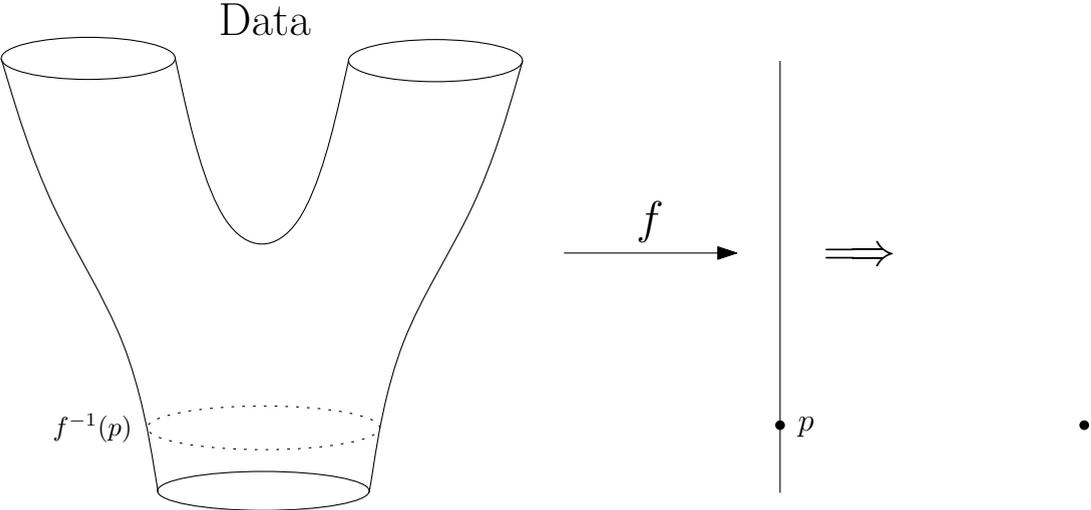
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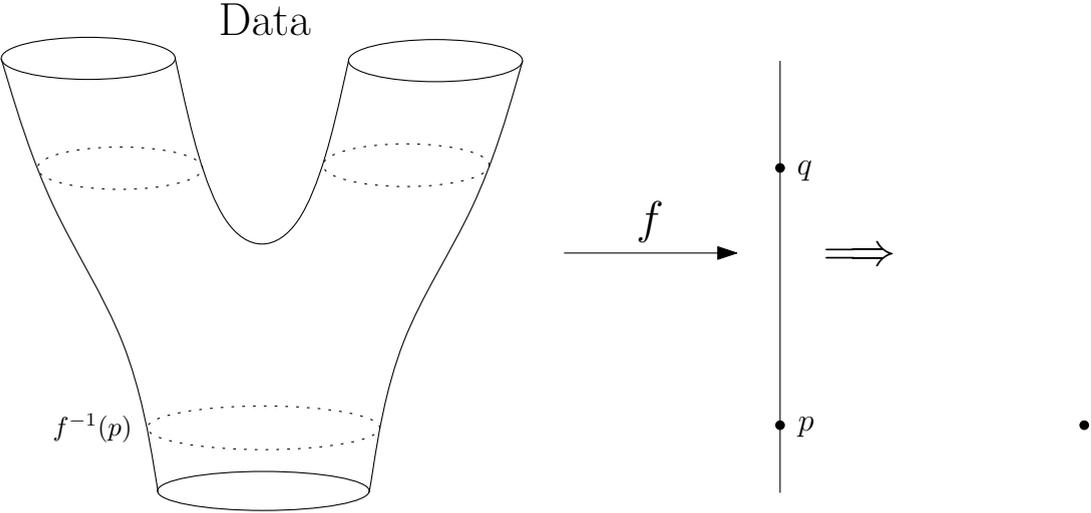
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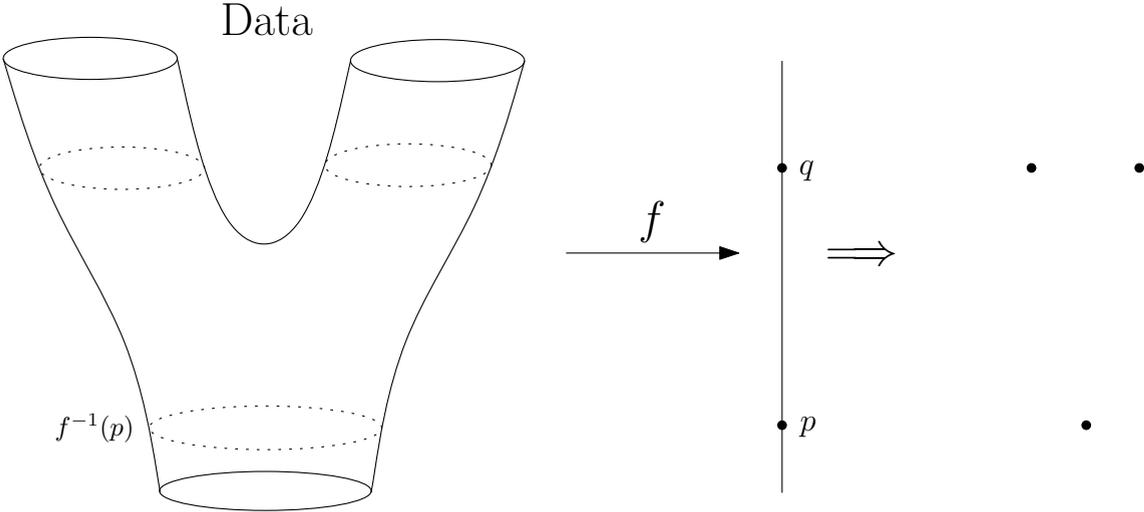
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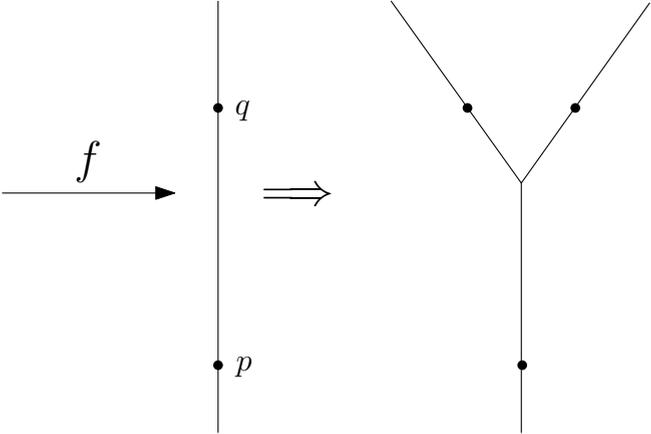
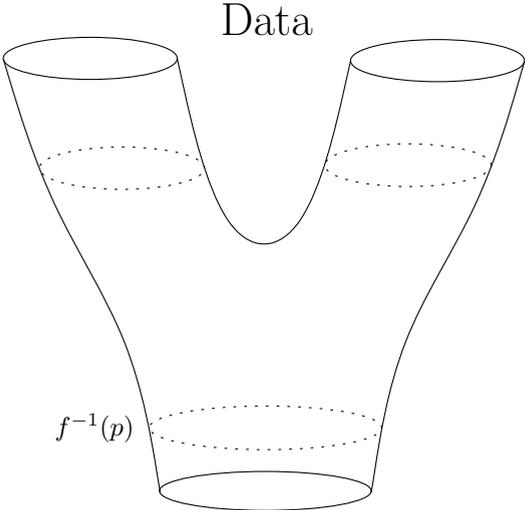
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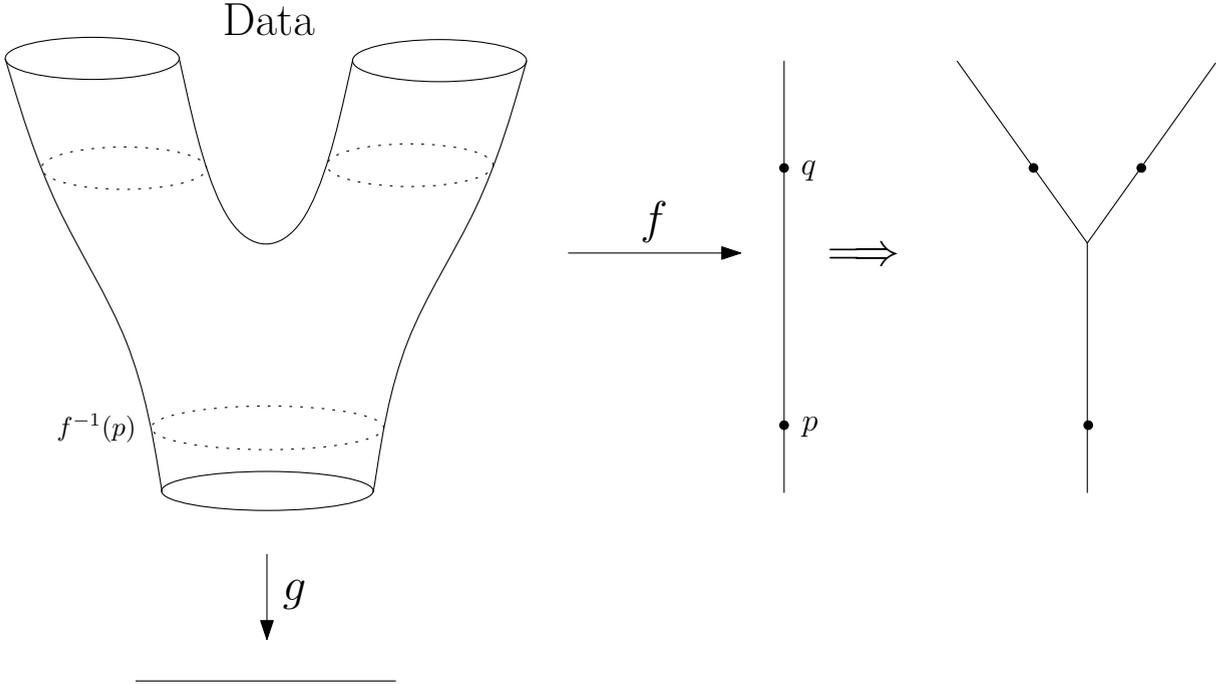
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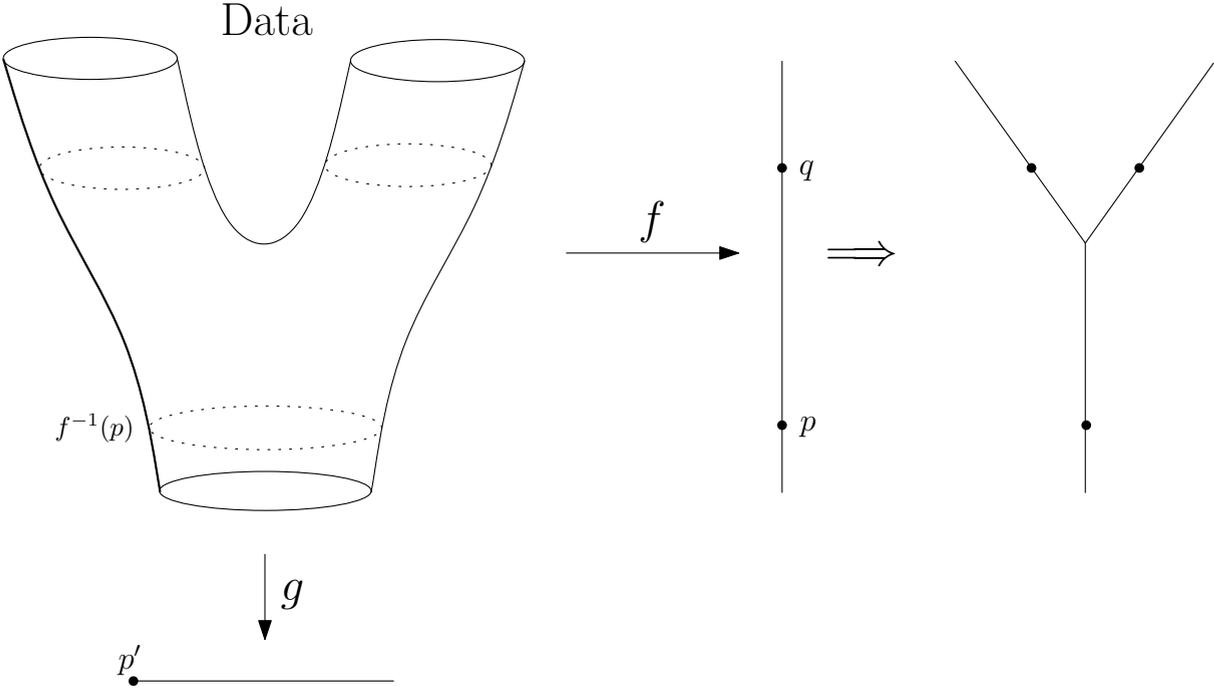
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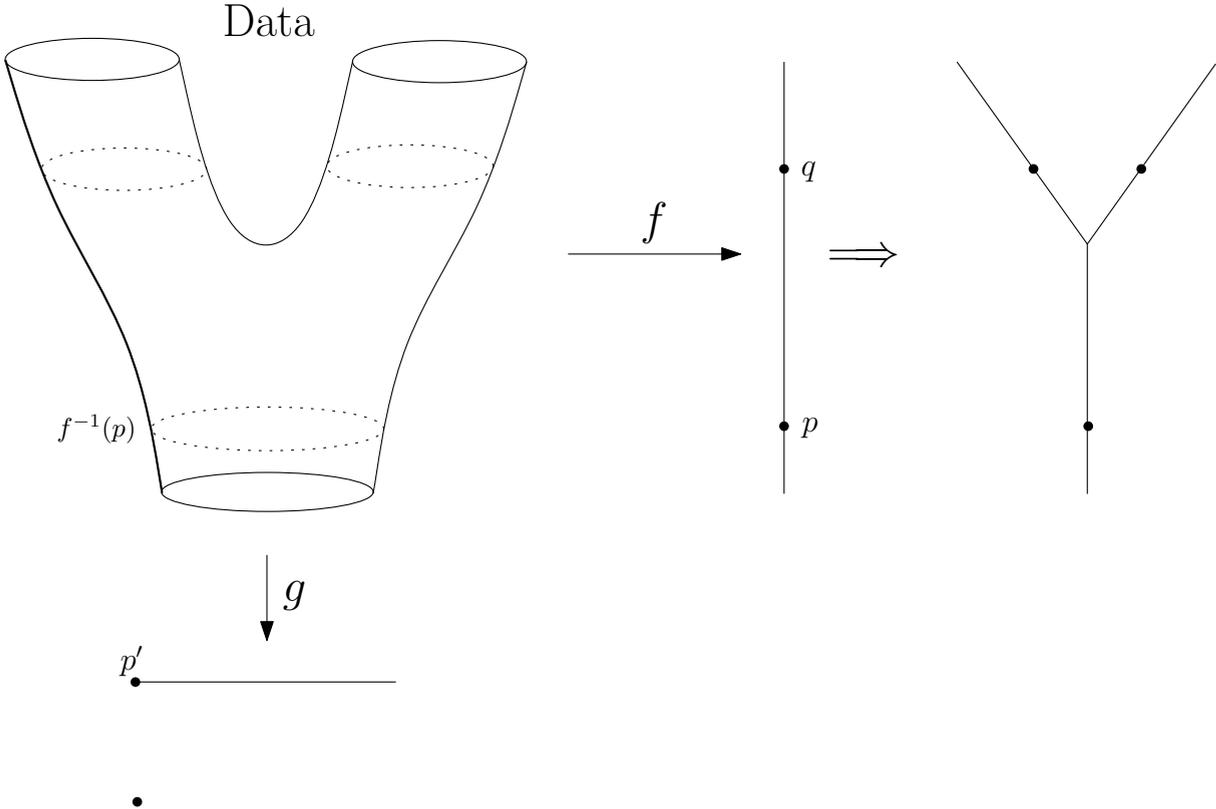
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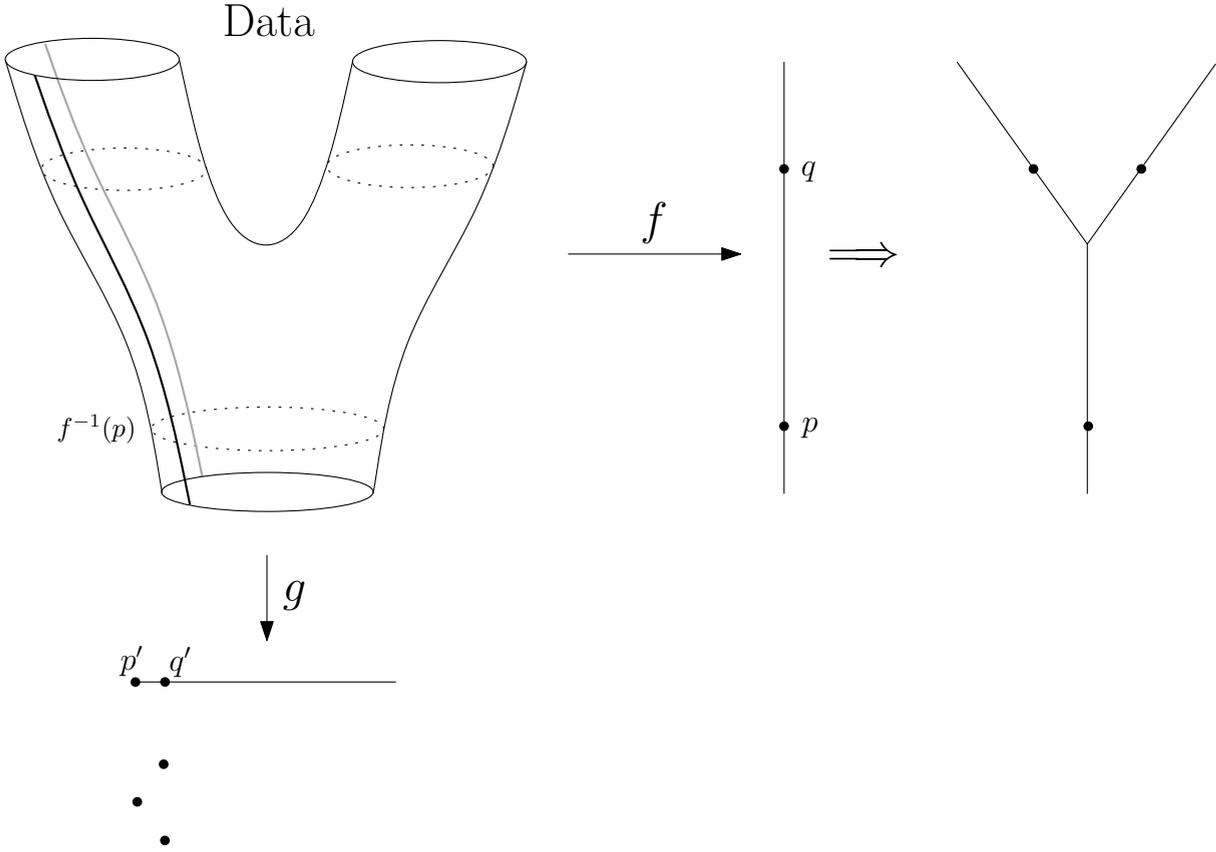
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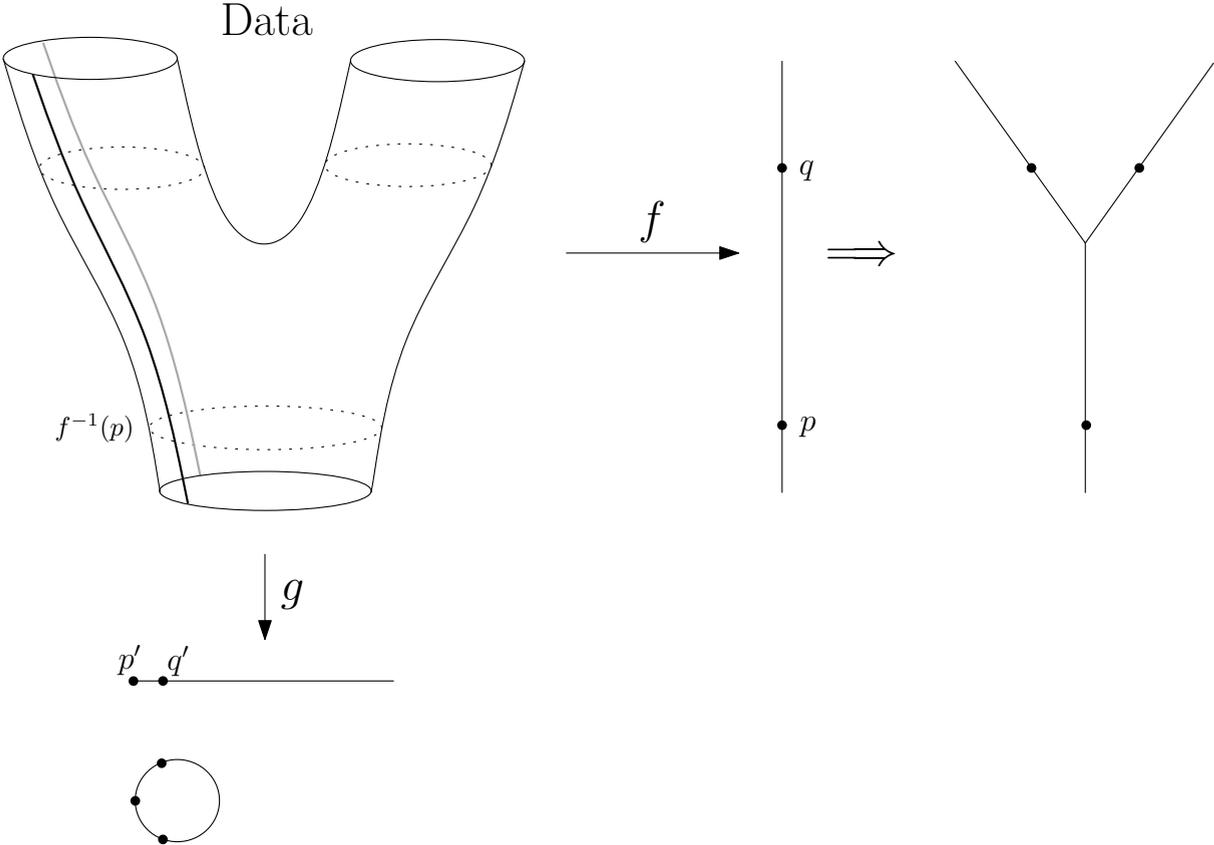
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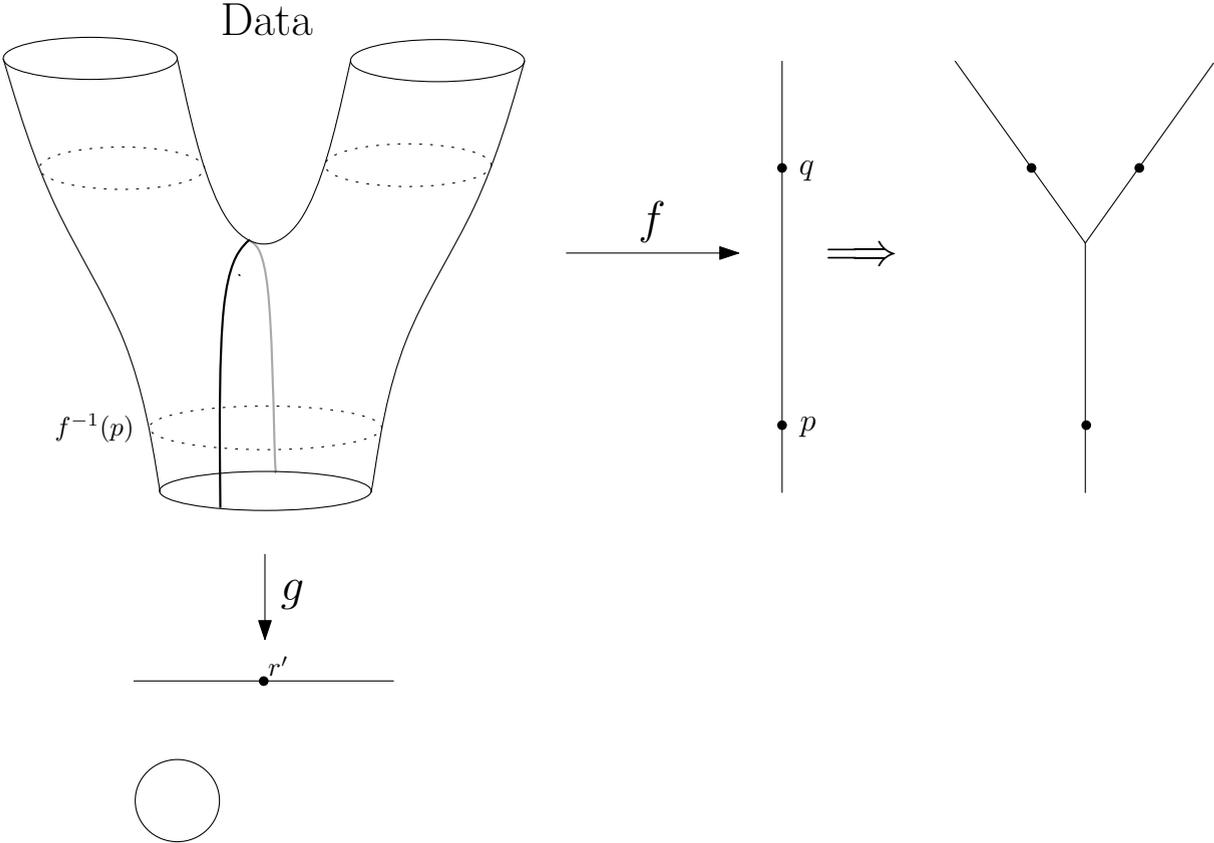
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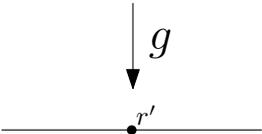
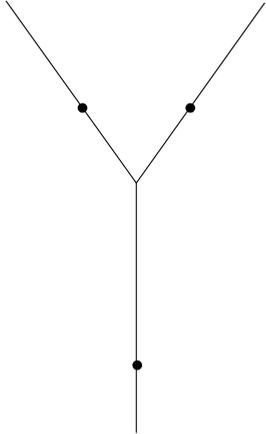
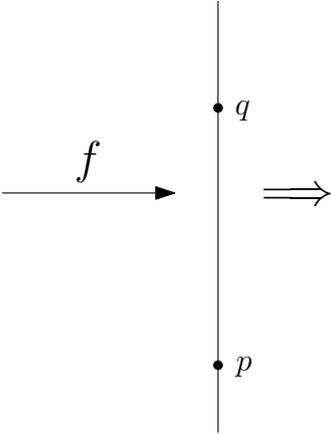
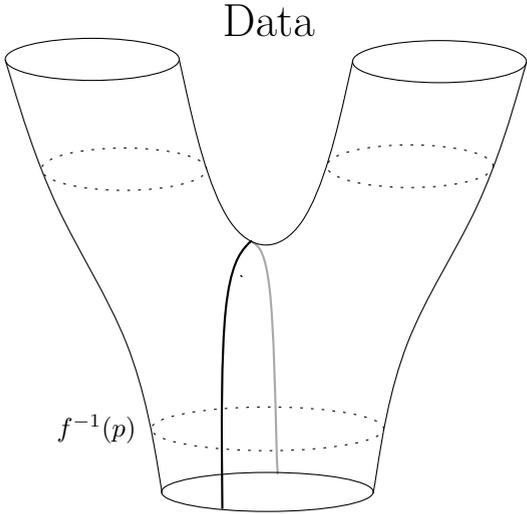
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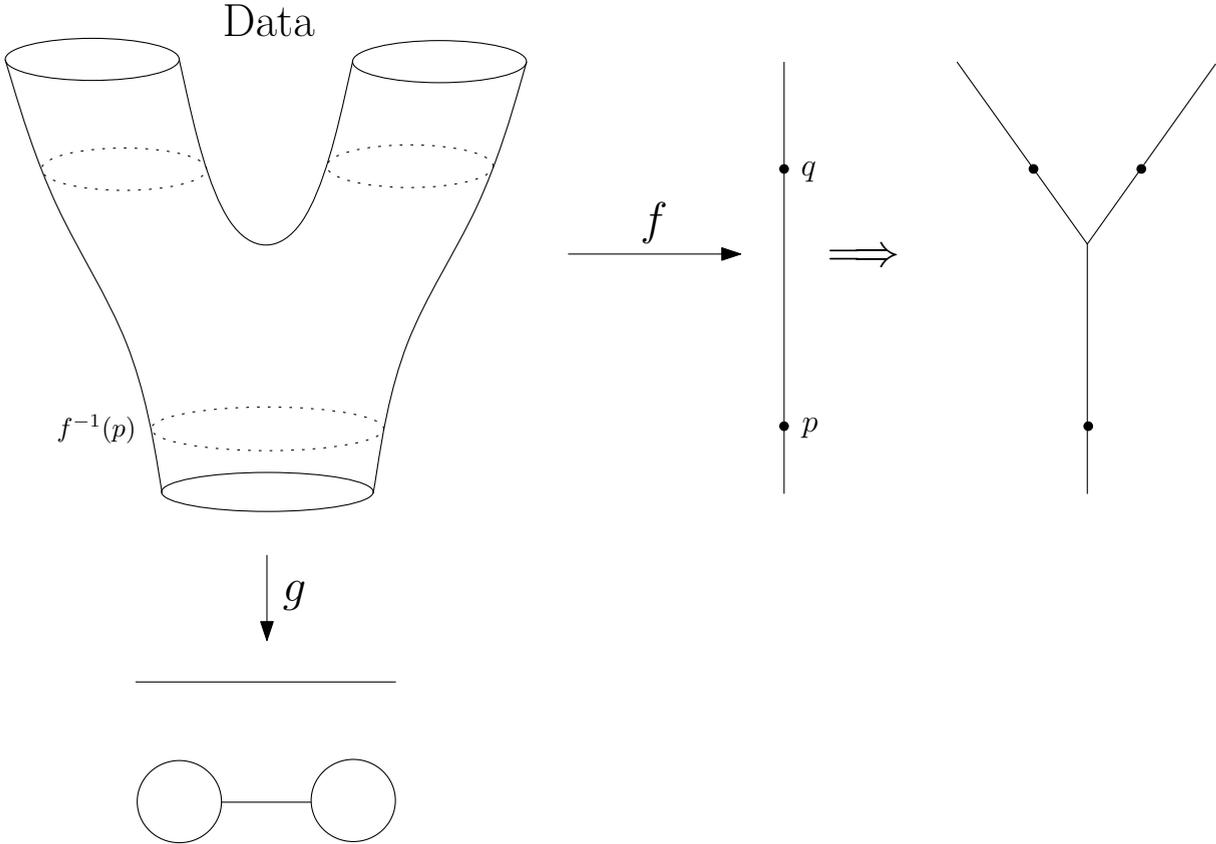
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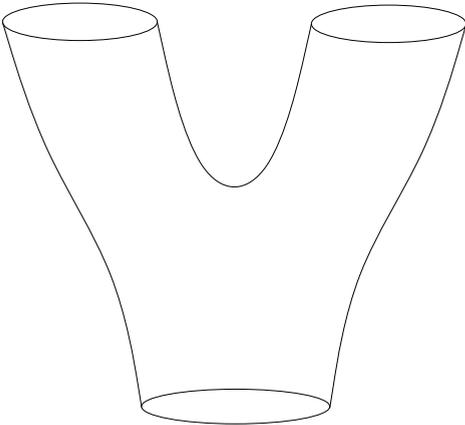


Why is this useful?

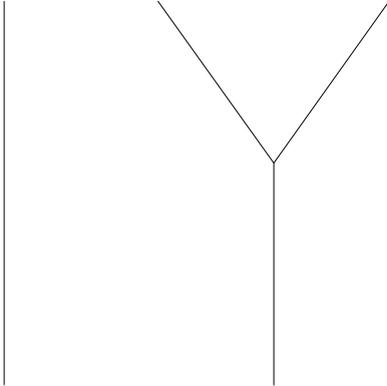
Why is this useful?

⇒ We get "easy" understanding of the localizations of quantities of interest.

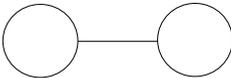
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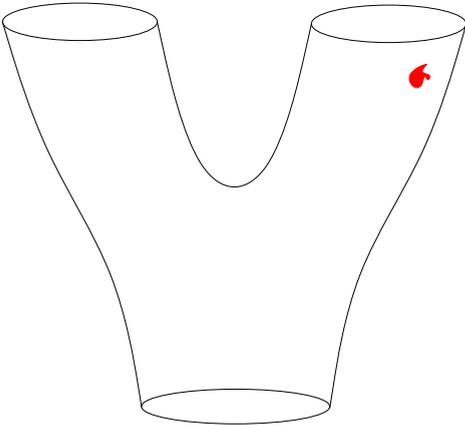
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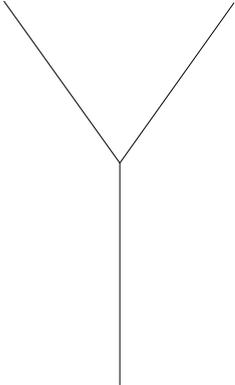
↓  $g$



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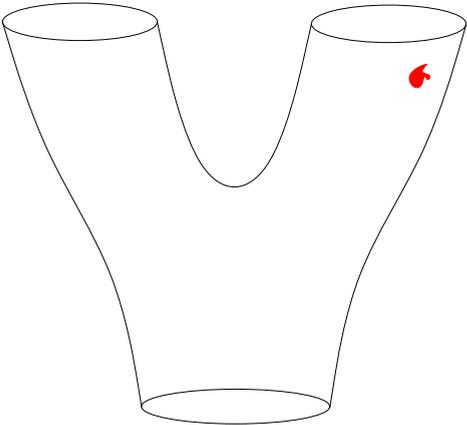
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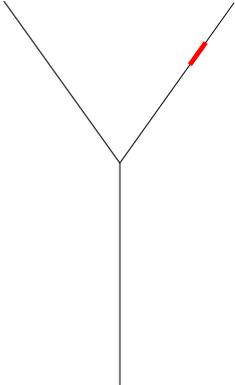
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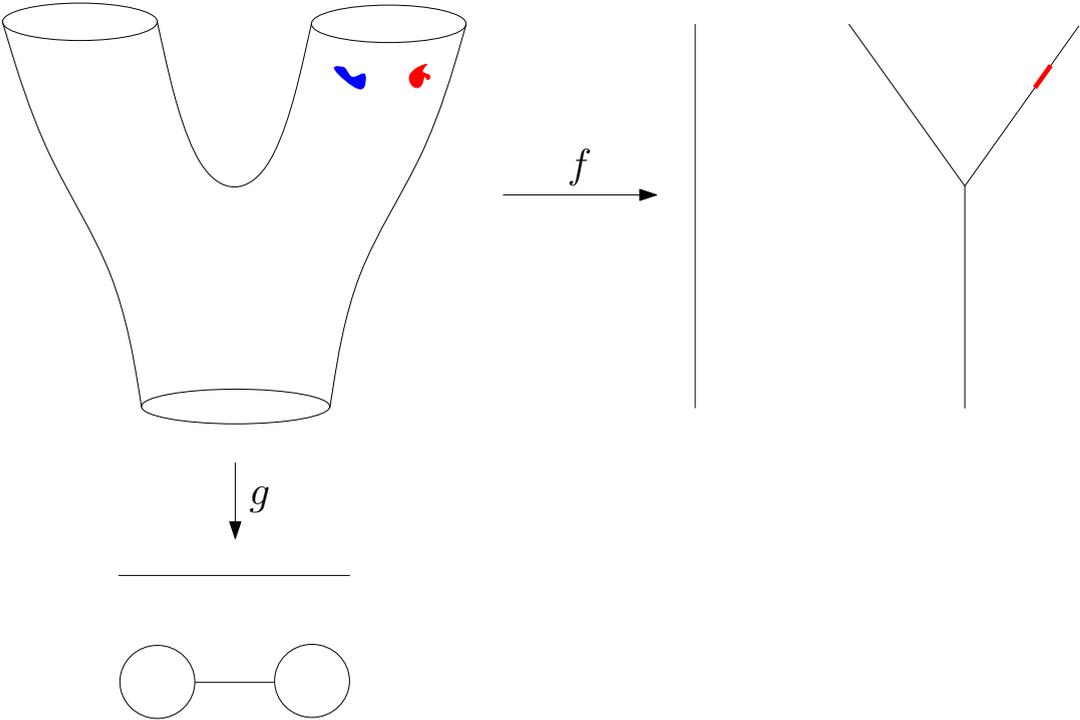
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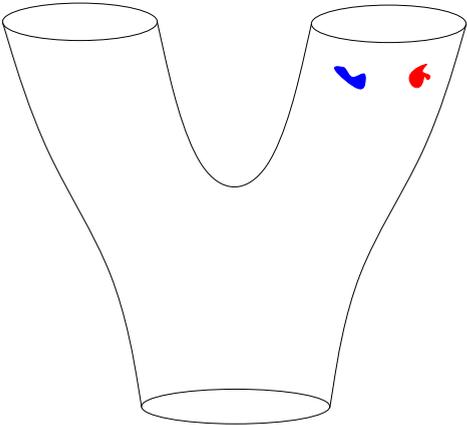
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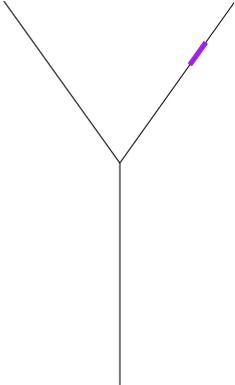
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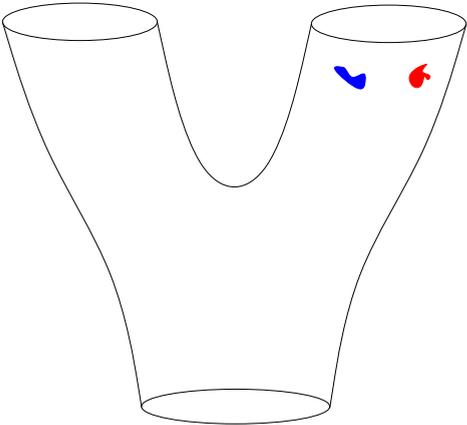
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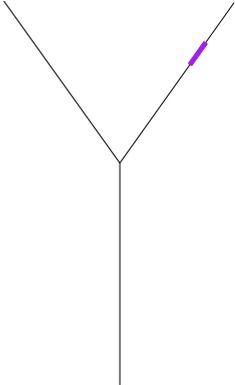
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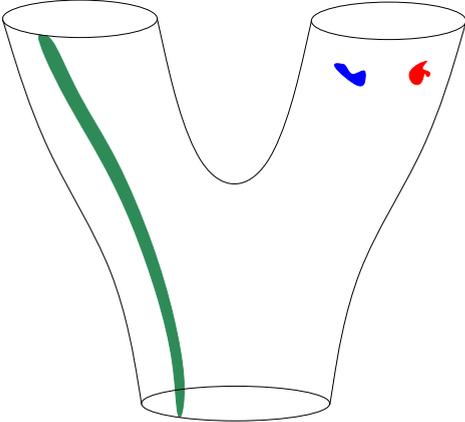
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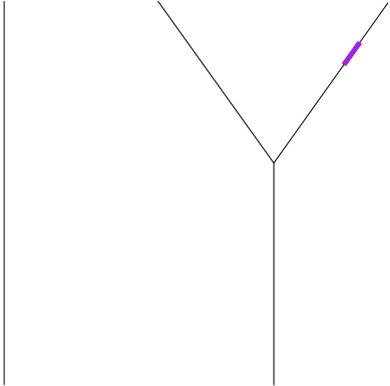
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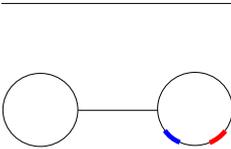
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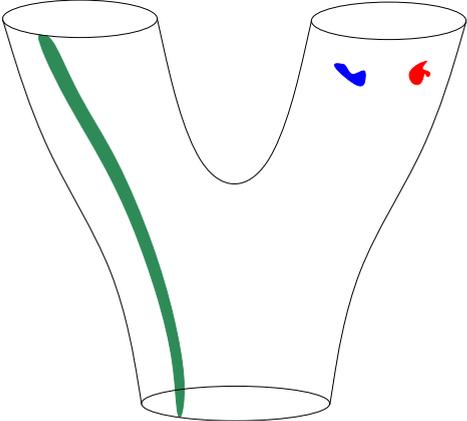
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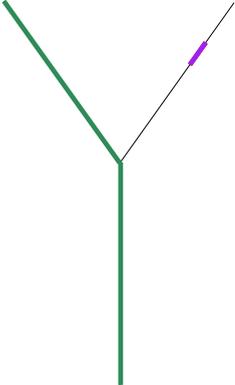
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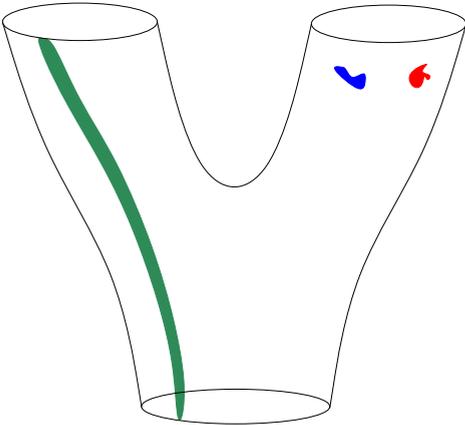
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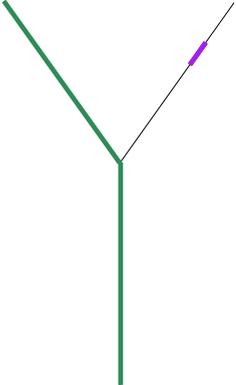
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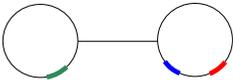
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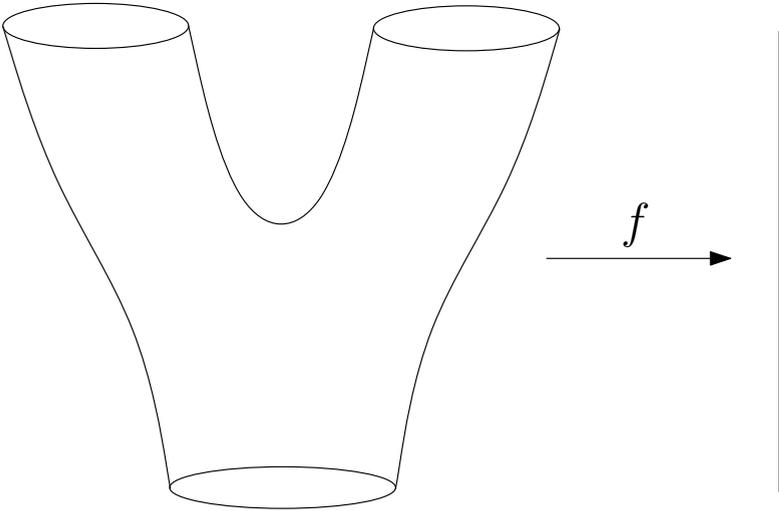
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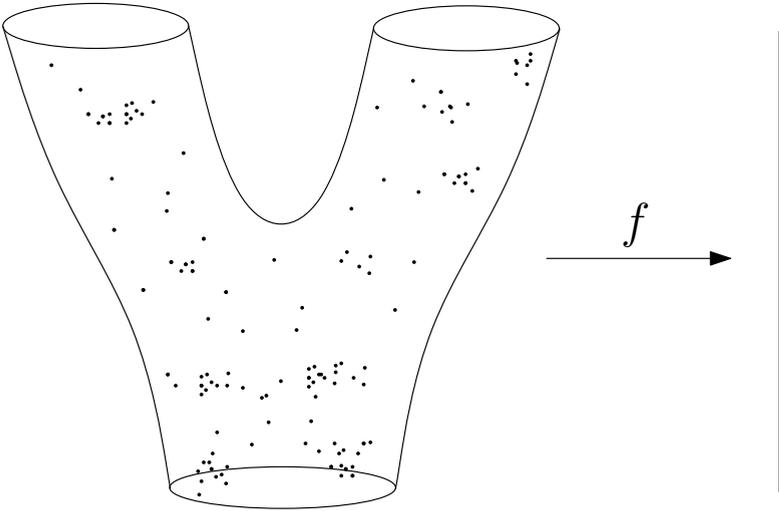
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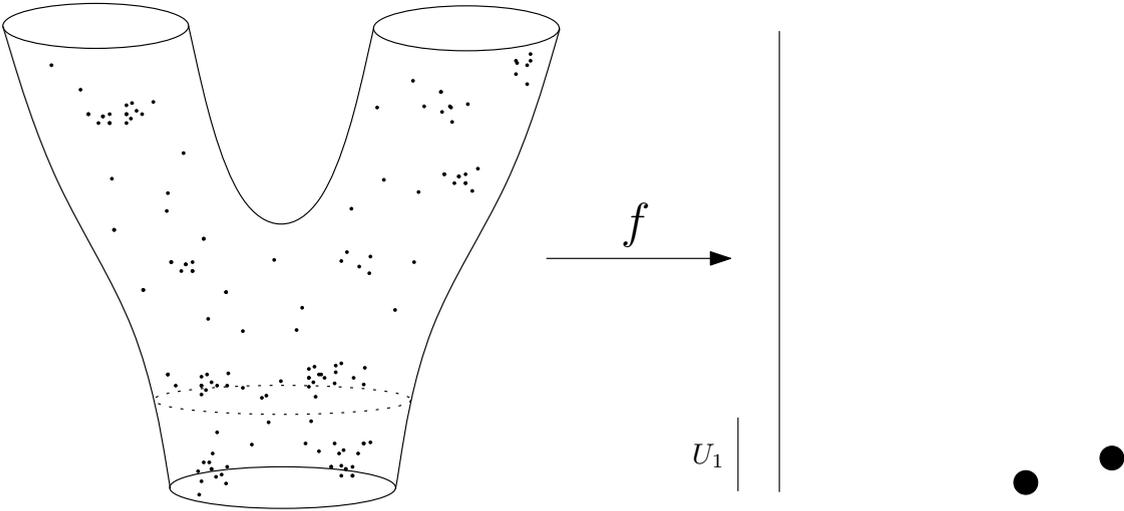
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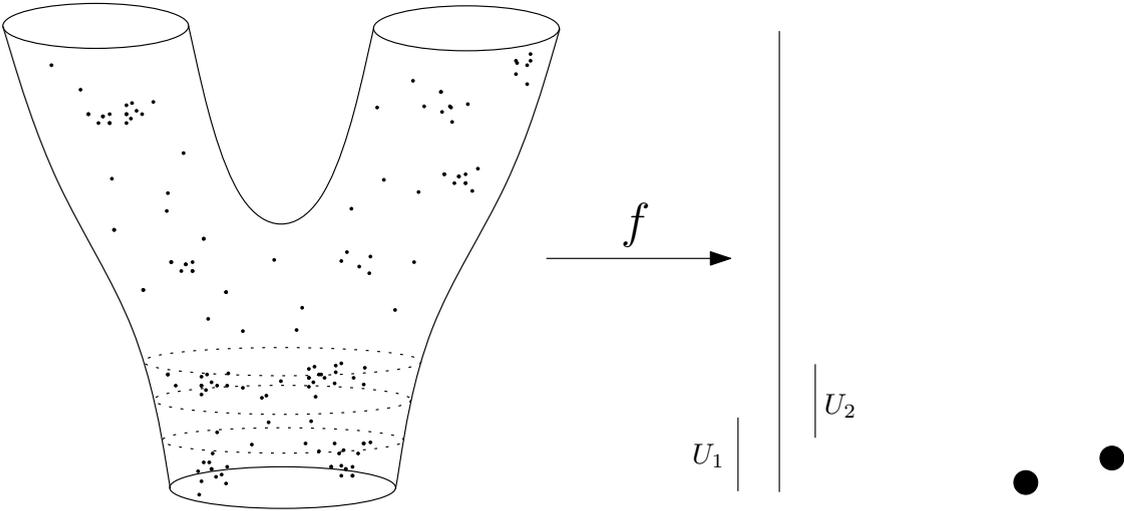
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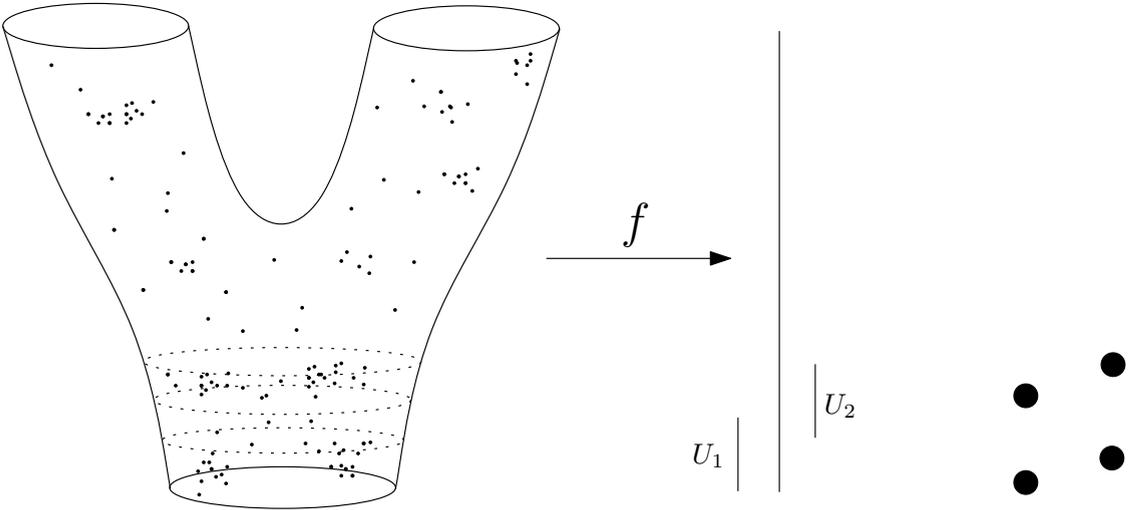
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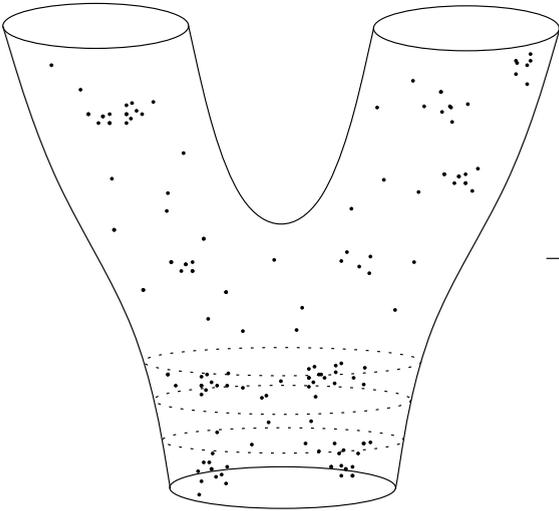
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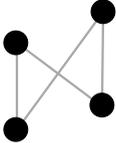
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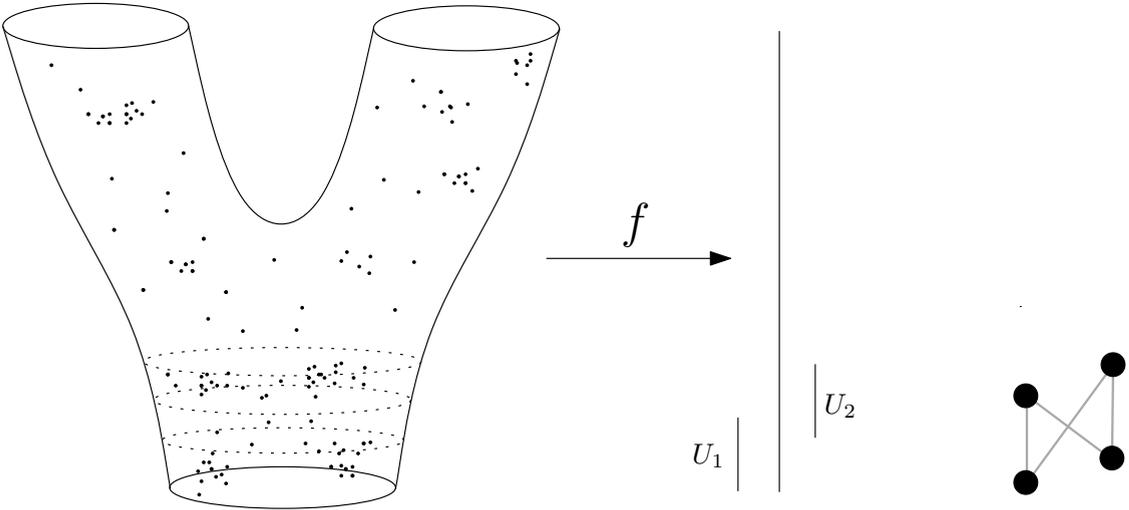
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$U_1$

$U_2$

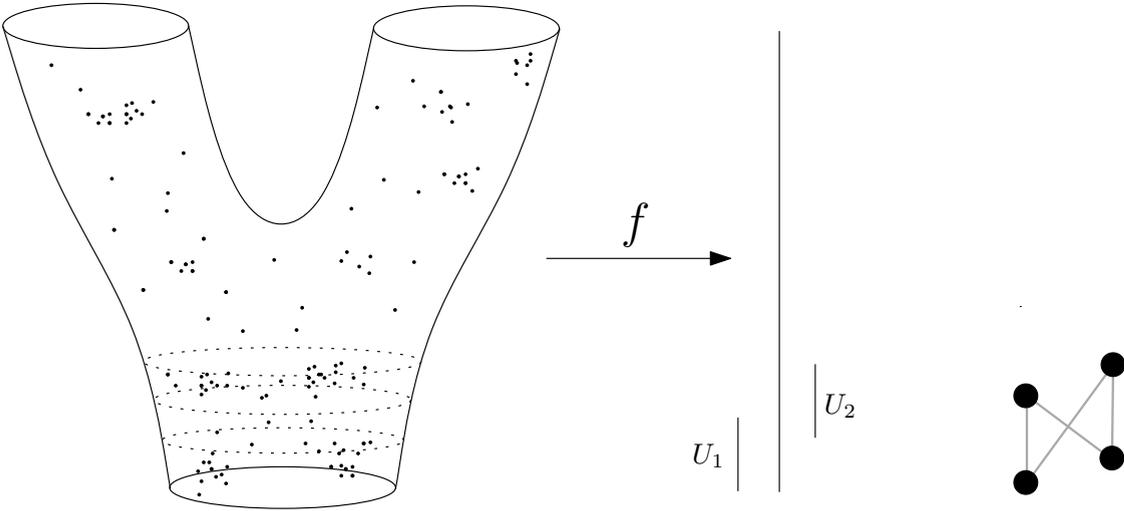


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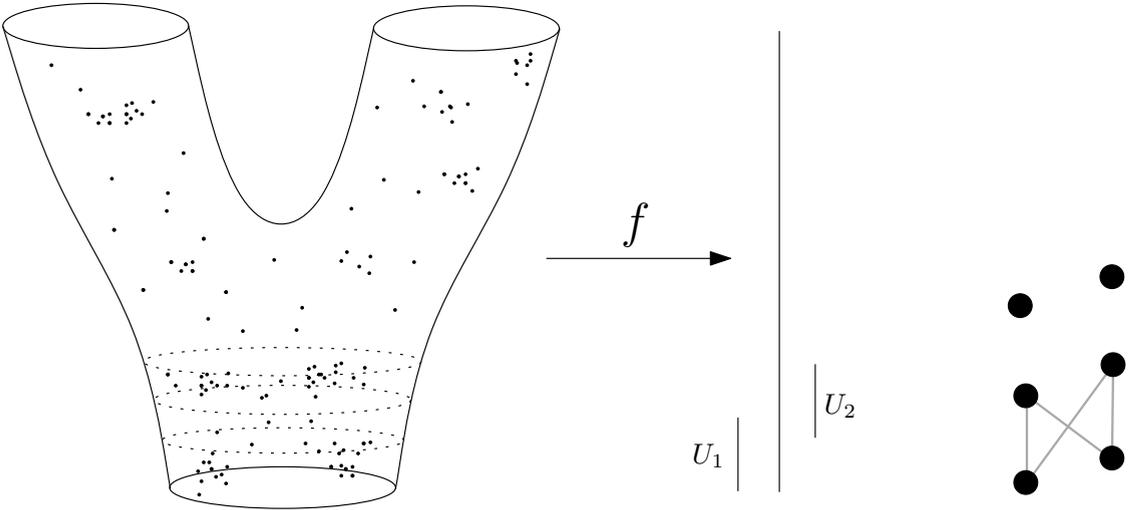
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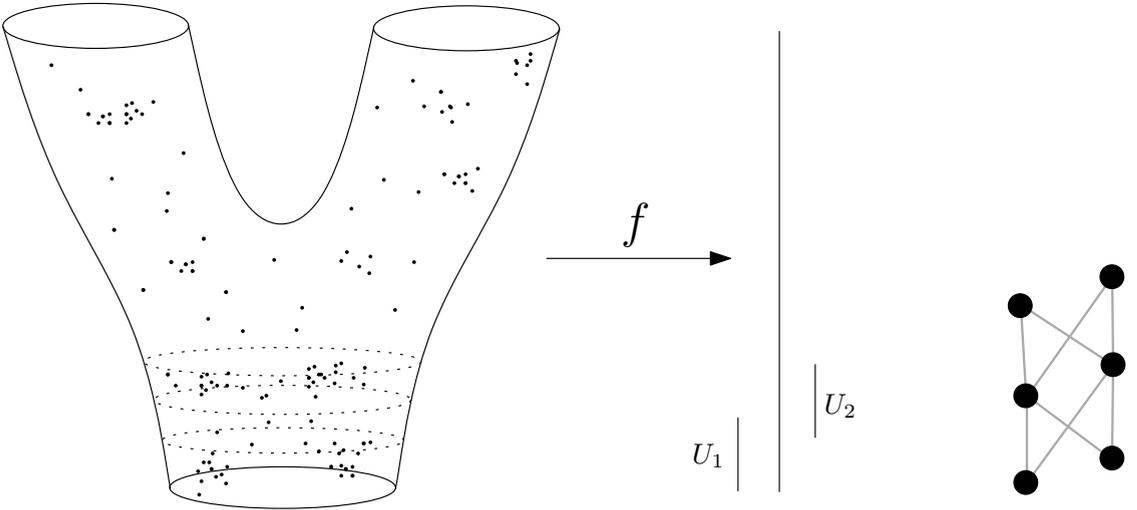
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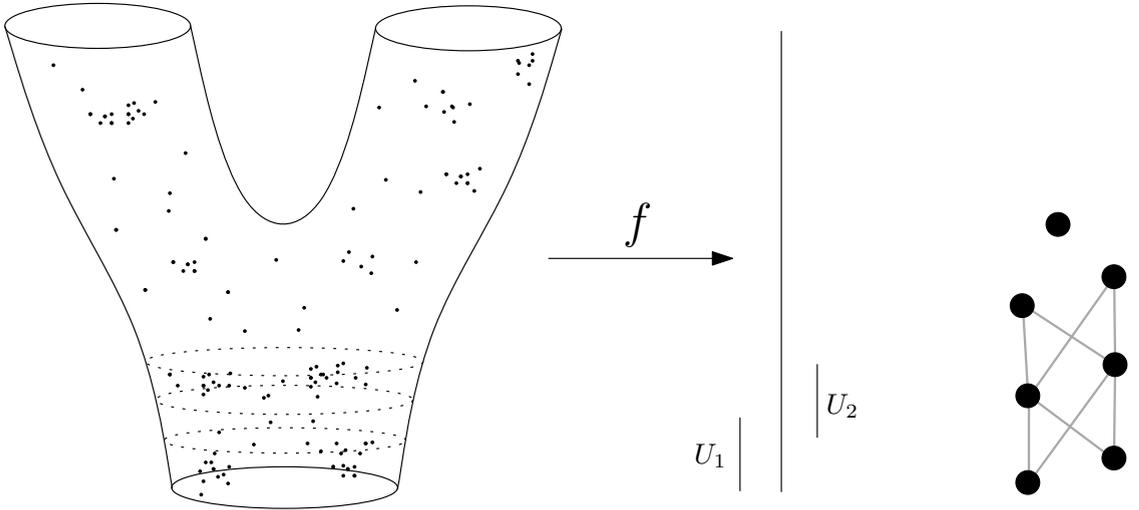
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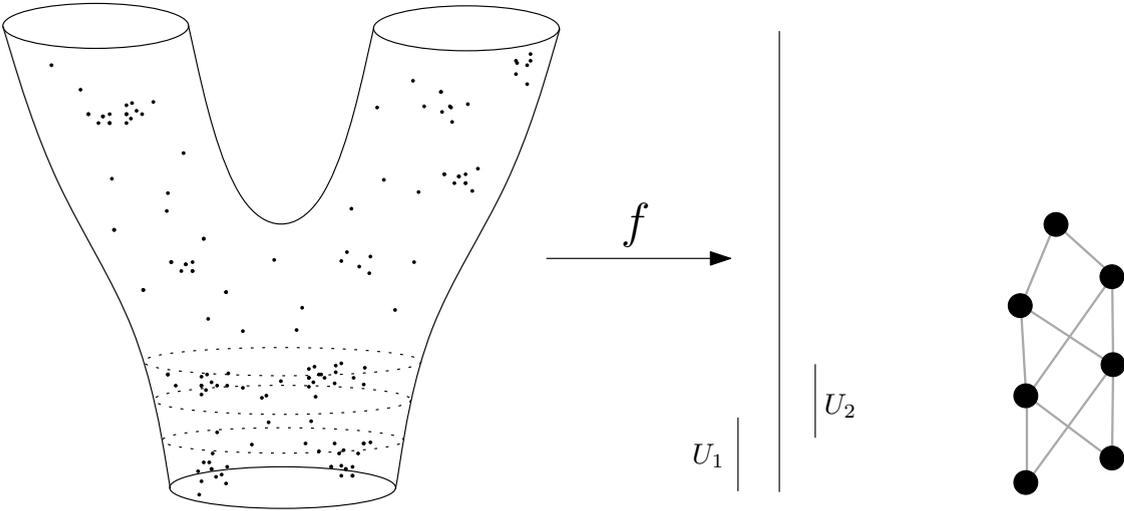
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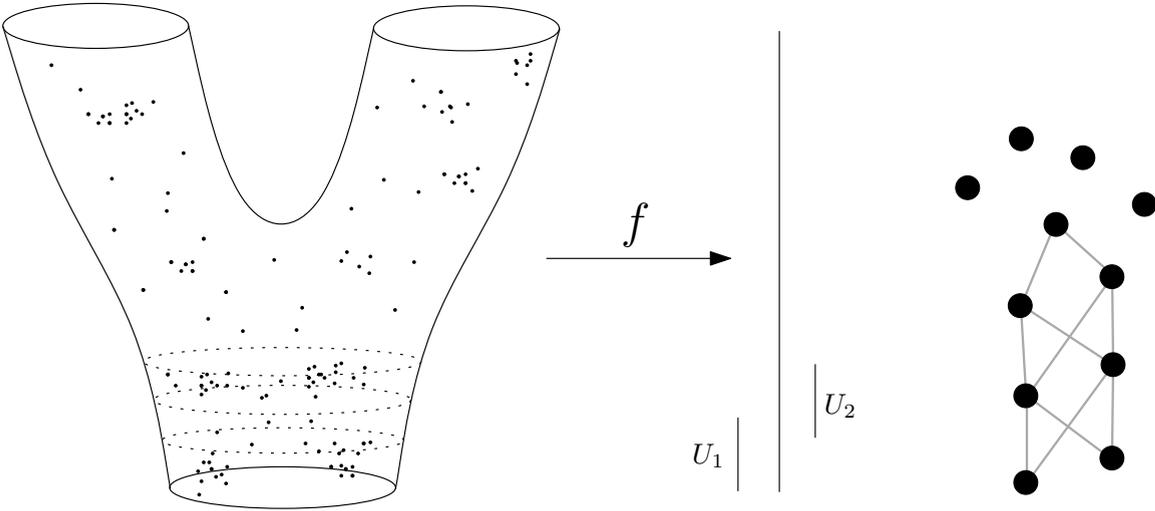
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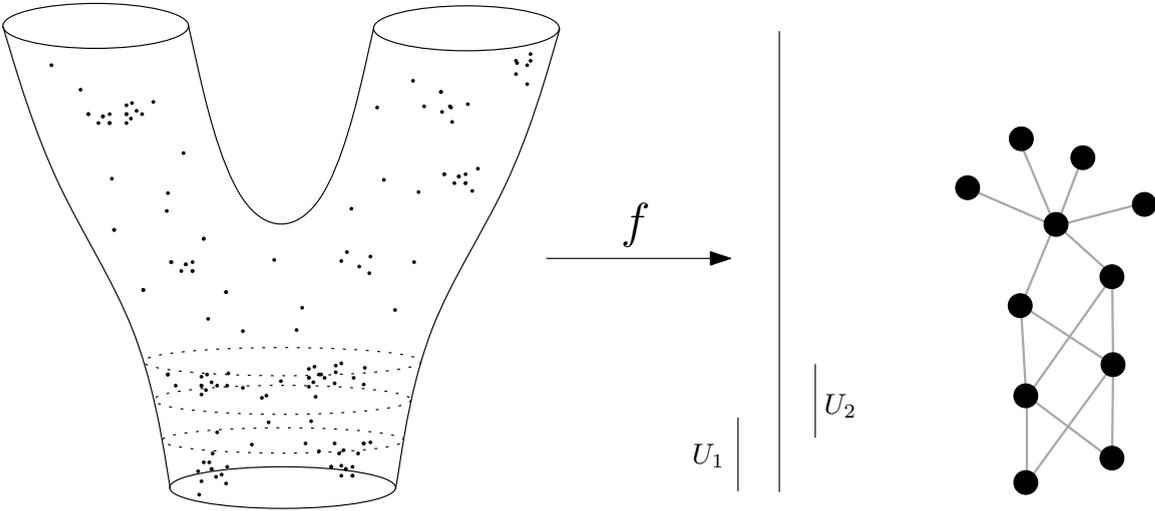
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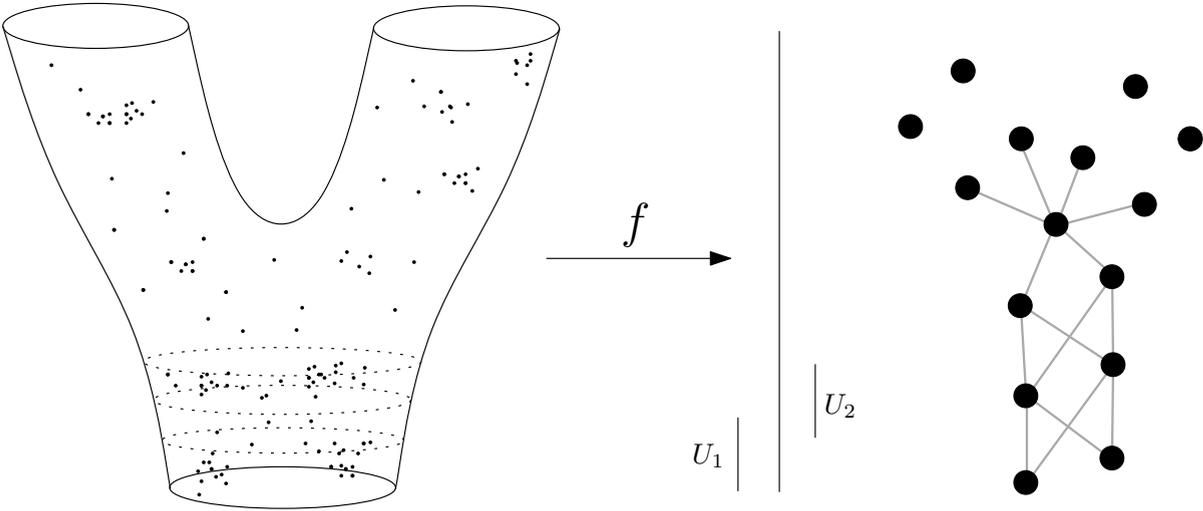
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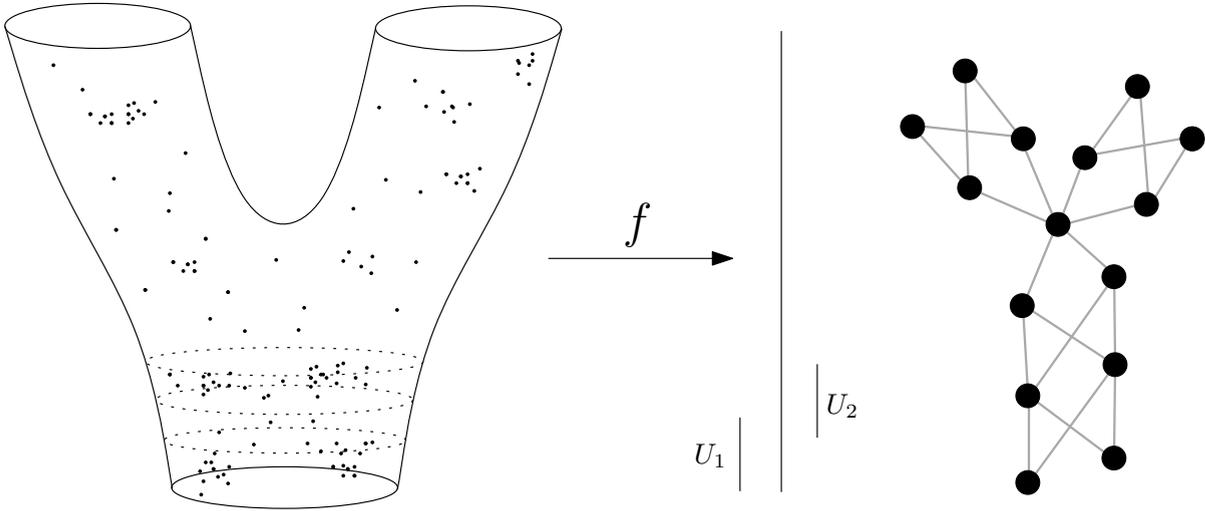
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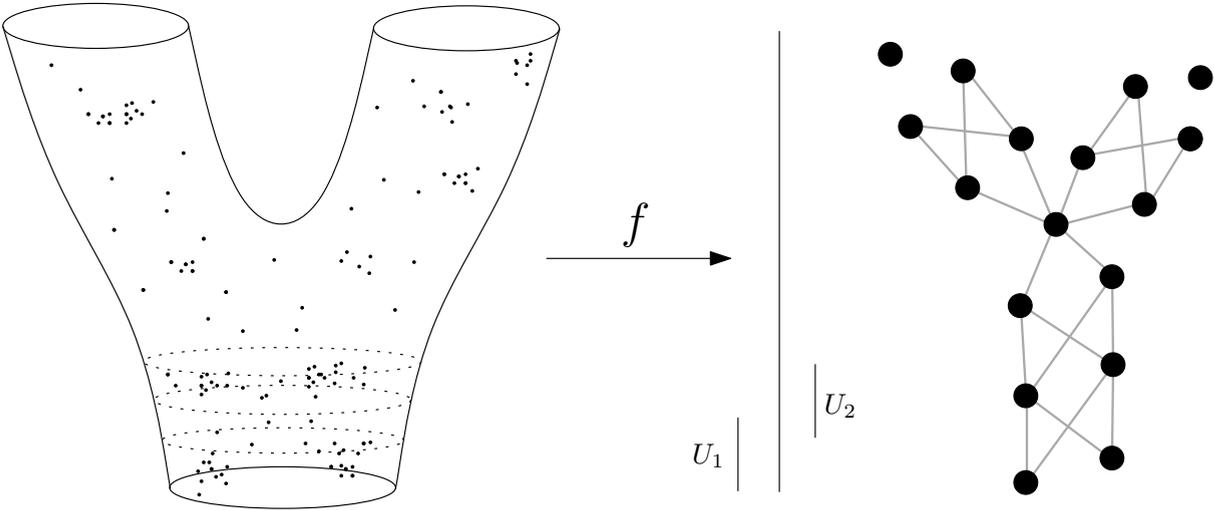
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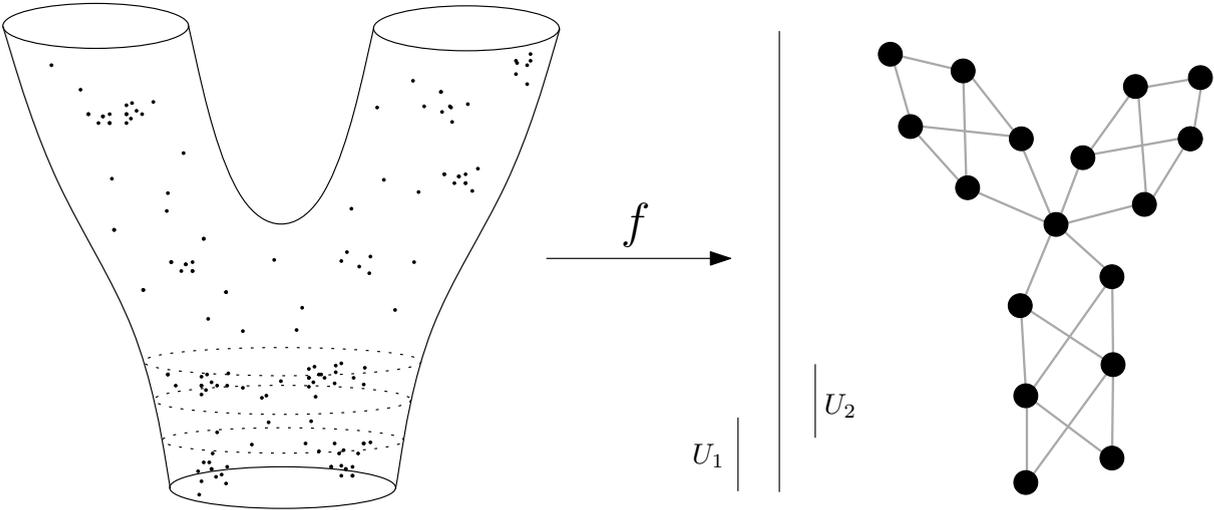
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Lenses: Where do they come from

The technique rests on finding good lenses.

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⇒ Luckily lots of people have worked on this problem

Lenses: Where do they come from

**A Non Exhaustive Table of Lenses**

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## Lenses: Where do they come from

- ▶ Standard data analysis functions

### **A Non Exhaustive Table of Lenses**

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Density

## Lenses: Where do they come from

- ▶ Standard data analysis functions

### **A Non Exhaustive Table of Lenses**

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Statistics

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- ▶ Standard data analysis functions
- ▶ Geometry and Topology

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Why use TDA?

Basic Example: Higher Fidelity PCA

## Unsupervised Learning: PCA

PCA is roughly speaking orthogonal projection onto the plane that best contains the data.

**Advantages:**

# Unsupervised Learning: PCA

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## **Advantages:**

- ▶ Provides unsupervised dimensionality reduction.

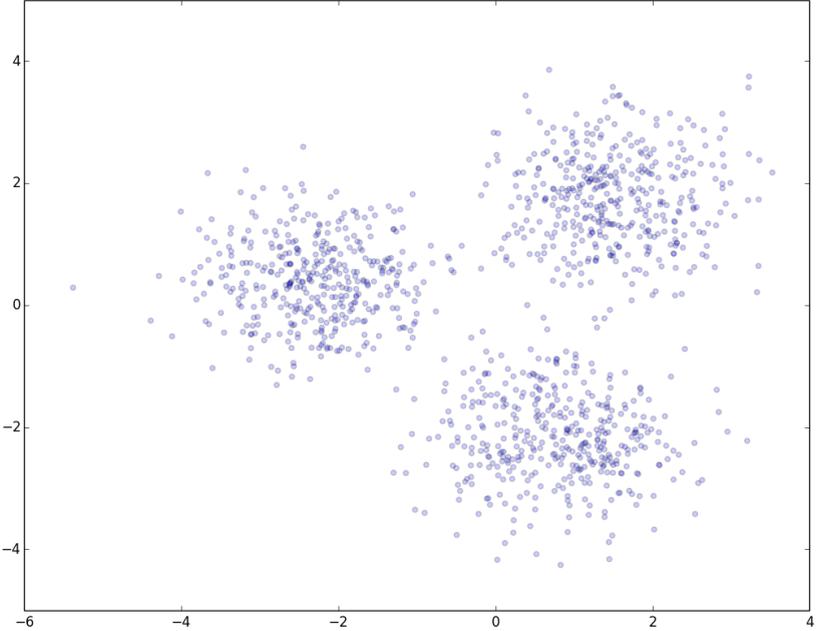
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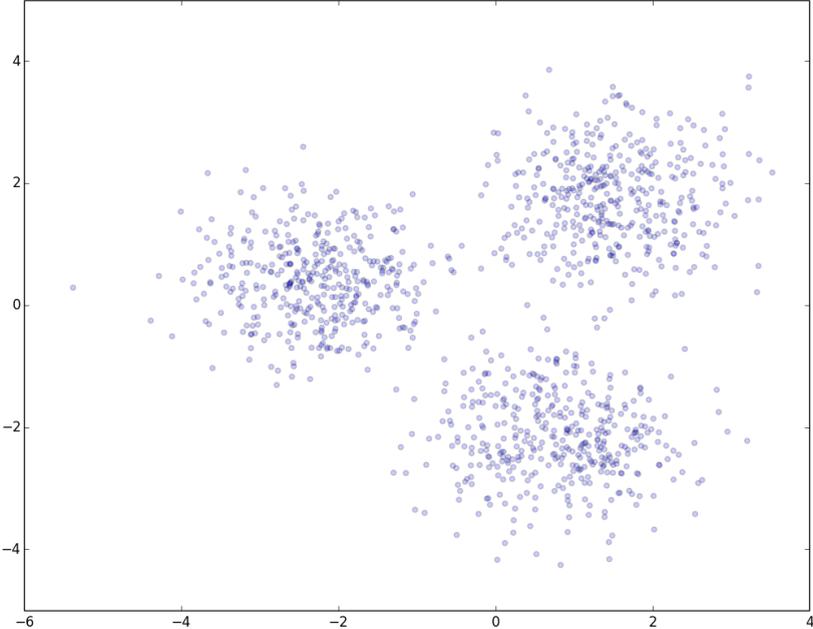
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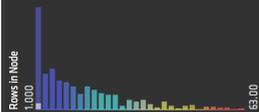
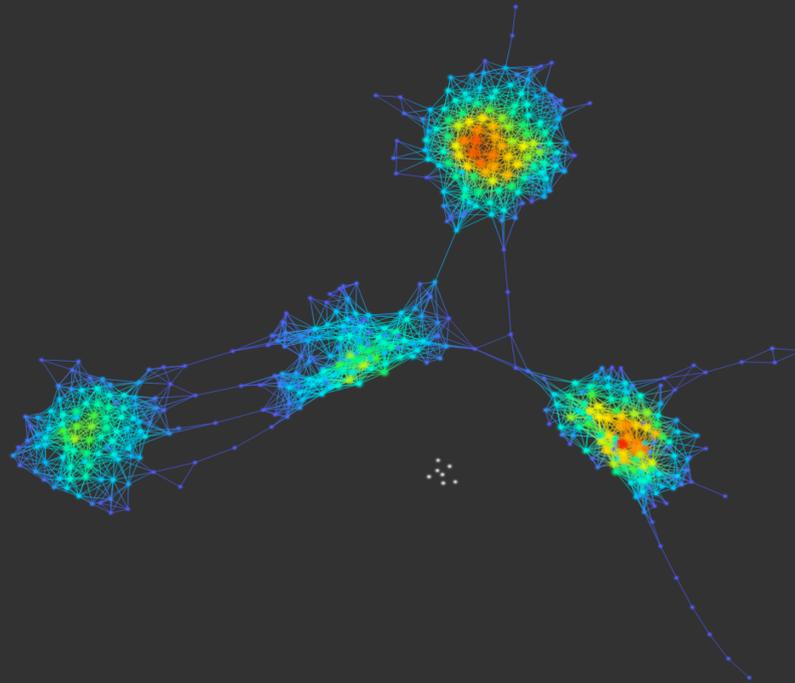


PCA captured 98.4% of the variance

# Unsupervised Learning: PCA

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As a *framework* for data analysis we get **higher fidelity** from existing tools.

## Real Examples

## Supervised Learning: Model Introspection

We can use TDA to examine what is happening with our machine learning models.

## Model Introspection: Outliers

**Data:** Customer attributes. Service usages, contractual details.

**Problem:** Customers commit fraud. Find customers with abnormal costs.

**Proposed Solution:** Create an ensemble of cost outlier models. Use these to flag customers as being fraudulent.

## Model Introspection: Outliers

**TDA Introspection:**

## Model Introspection: Outliers

### **TDA Introspection:**

- ▶ Create a dataset that contains all non-cost information.

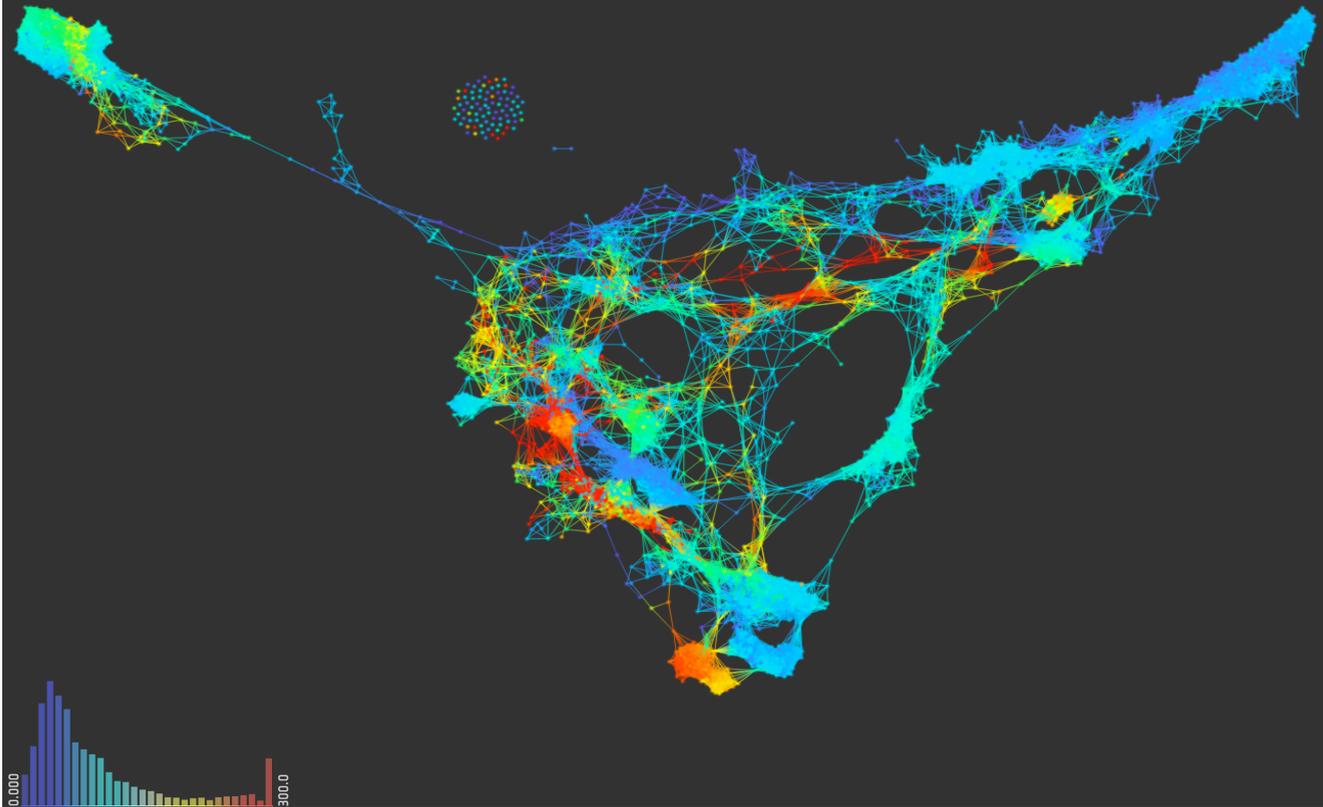
## Model Introspection: Outliers

### **TDA Introspection:**

- ▶ Create a dataset that contains all non-cost information.
- ▶ Color by who is being flagged by the ensemble as being a (high) cost outlier.

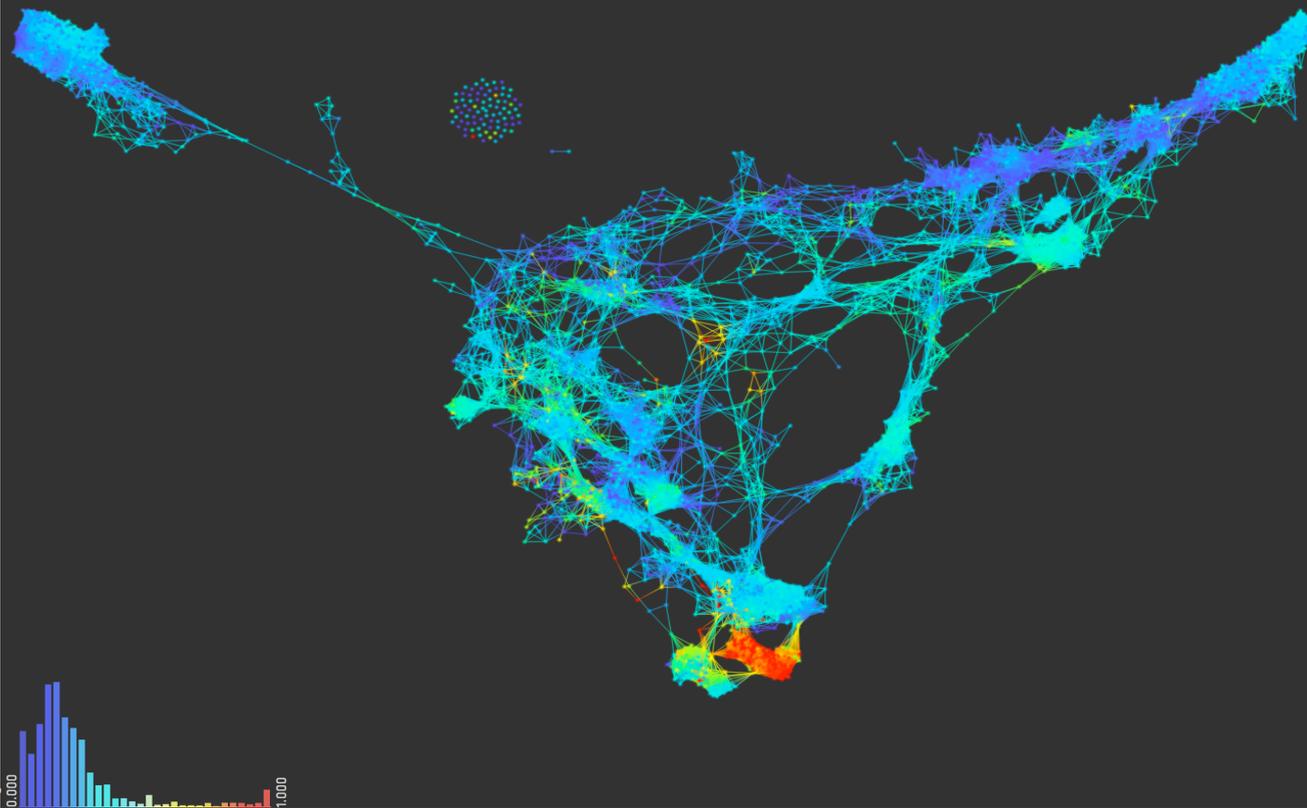
# Model Introspection: Customer Cost

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# Model Introspection: Model Outliers

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## Model Introspection: Outliers

Observation:

- ▶ The different (independent?) models are all flagging the same group of customers as cost outliers.

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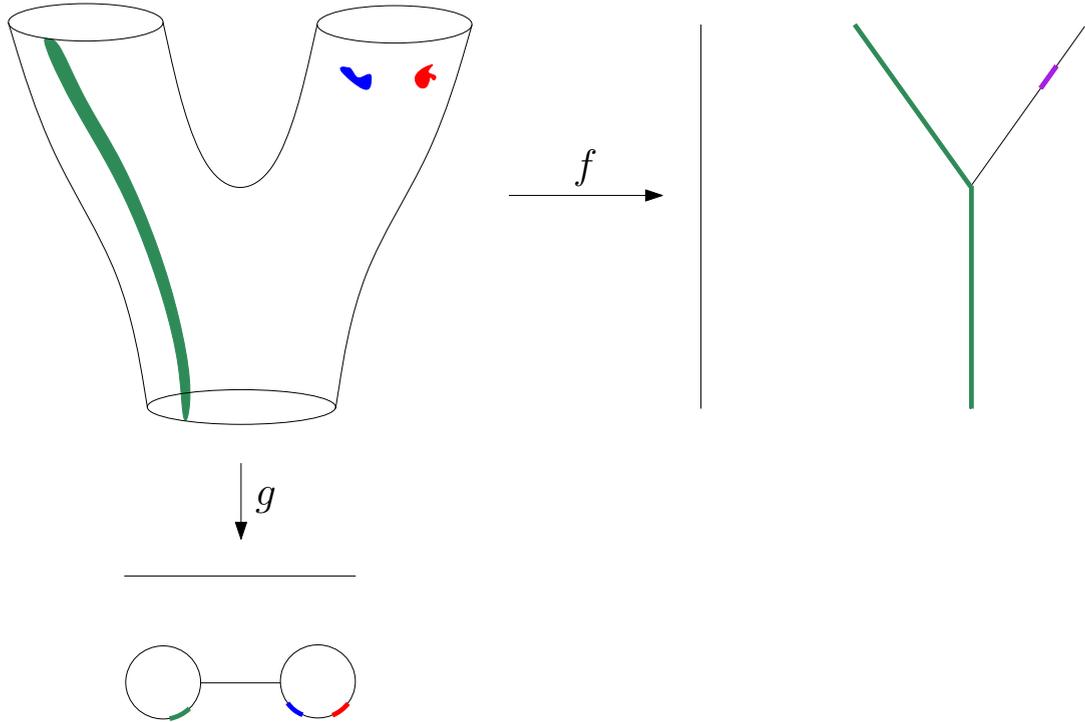
⇒ TDA tells us where to look in our data for problems and questions.

Thank You!

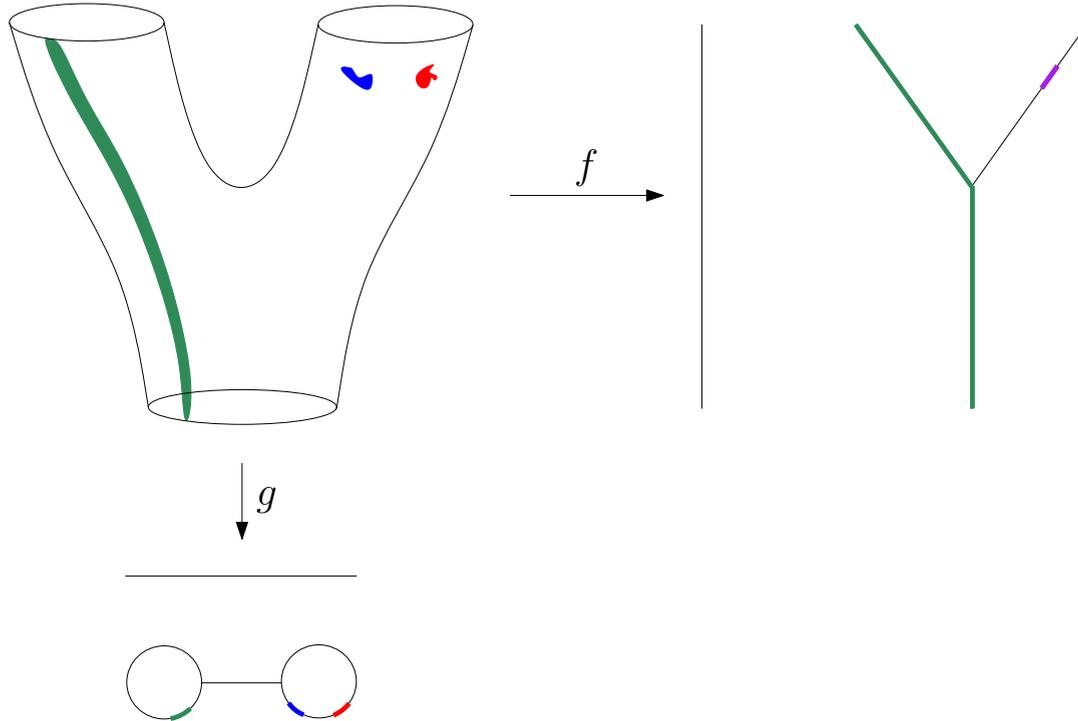
<http://www.ayasdi.com/>

## Part II

1. Review
2. Why Topology? (With Examples)
3. More Applications



TDA is a machine for creating geometric/topological summaries.



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The shape (segmentations, groupings, features) represent verified hypothesis. You have to decide if they are interesting.

## Why Topology?

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We'll examine them in turn.

## 1) Coordinate Invariance

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⇒ You want to study properties of your data that are invariant under coordinate changes.

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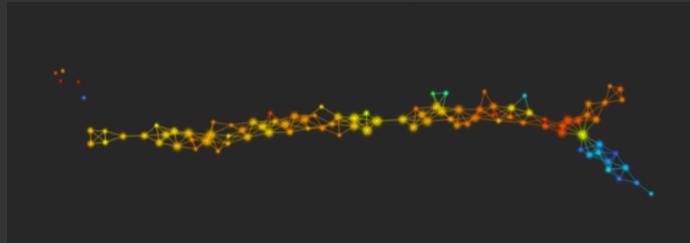
- ▶ Samples from different patient populations
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⇒ Different coordinates on Cancer

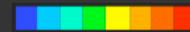
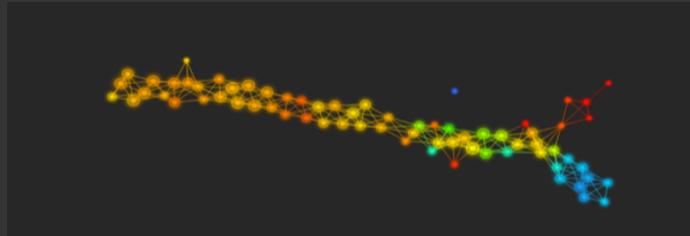
# Coordinate Invariance: Gene Expression

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GSE230



ESR1 Levels

## 2) Deformation Invariance

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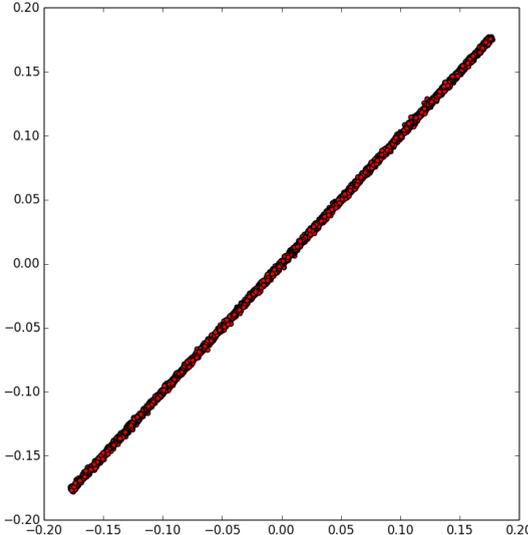
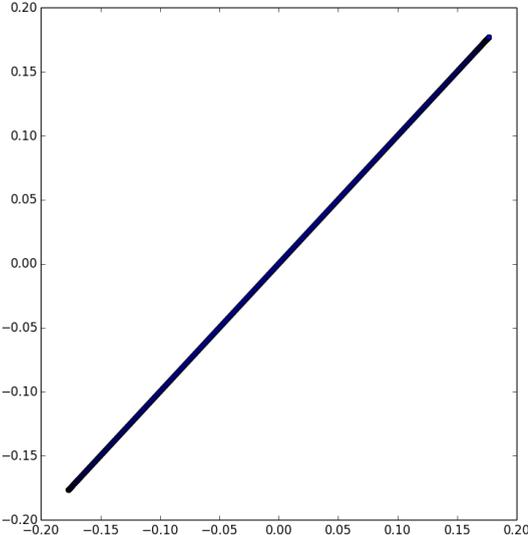
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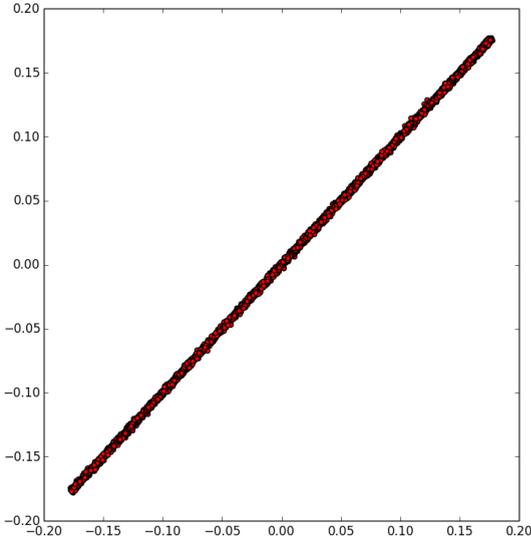
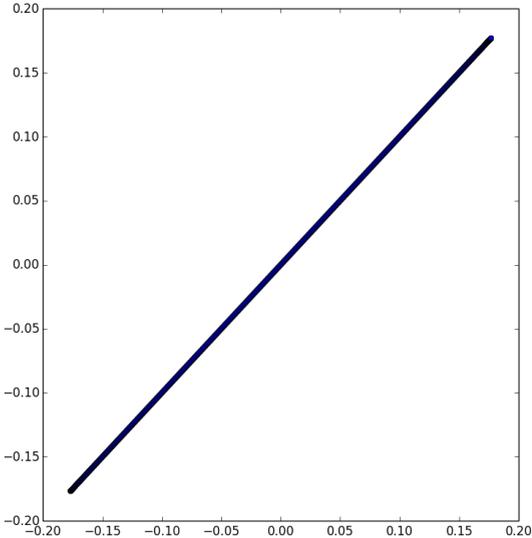
**Advantage:** Makes problems easier.

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- ▶ Less preprocessing of the data.
- ▶ Robust (stable) answers.

# Deformation Invariance: Line and Noisy Line



# Deformation Invariance: Line and Noisy Line

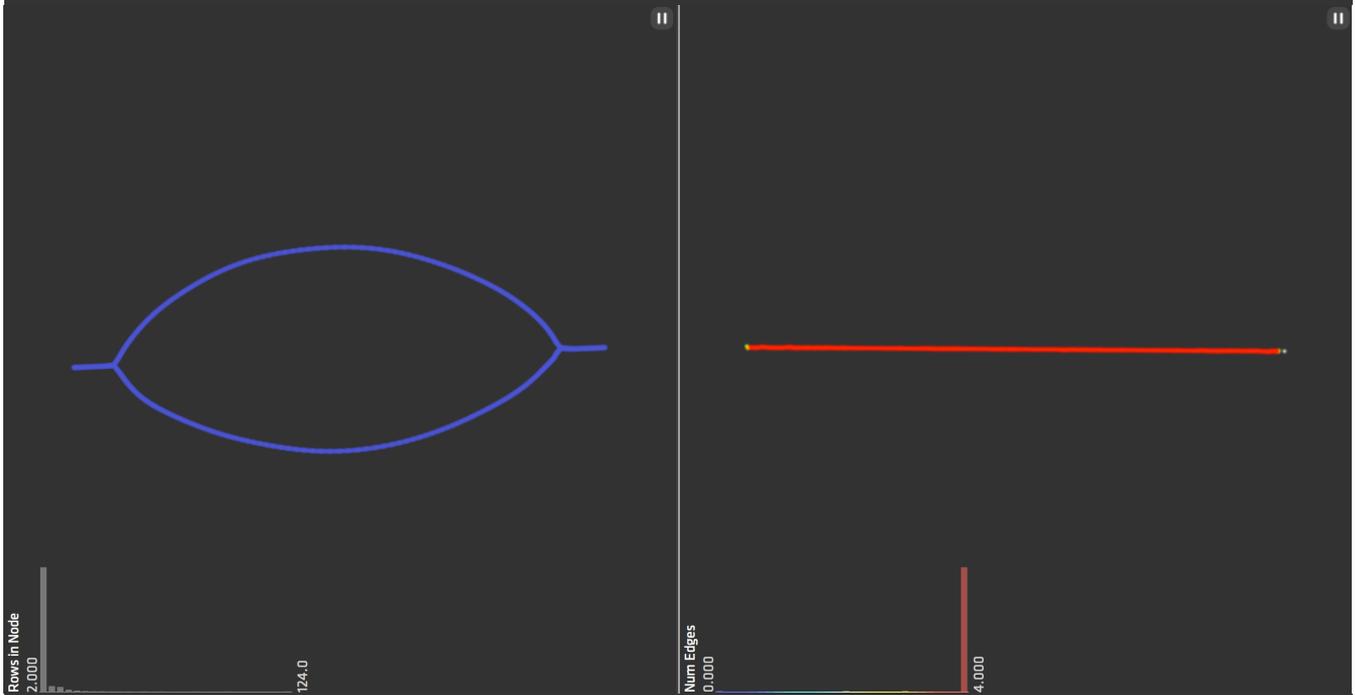


Pearson Correlation: 0.999998 resp 0.9999

Use x-axis coordinate as a lens. Expect that we will get two lines out.

# Deformation Invariance: Line is a Circle

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## Deformation Invariance: Circle and Noisy Line

Some lessons.

- ▶ We can be surprised even when we think the solution is obvious. Both examples had almost perfect correlation.

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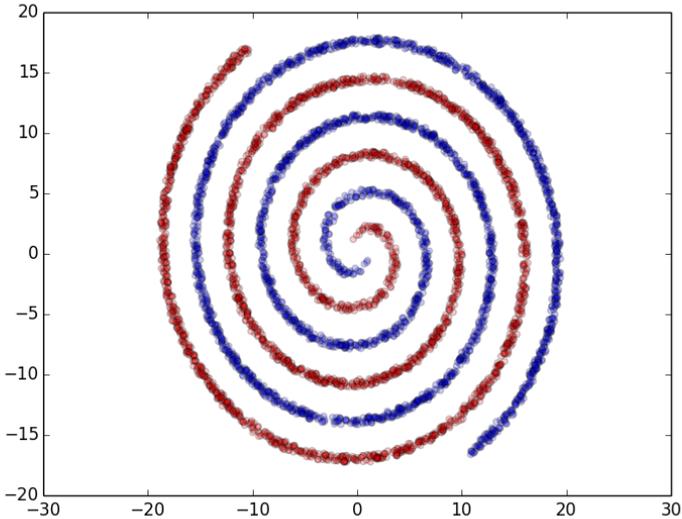
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- ▶ We **did not** find structure in noise.

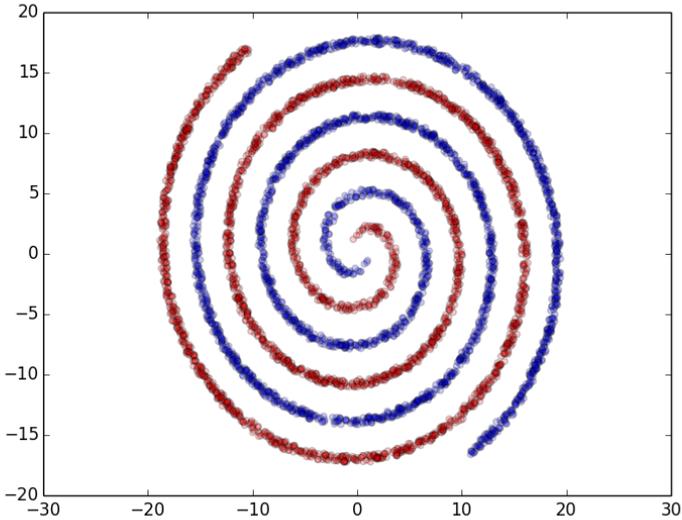
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Separate the two classes.

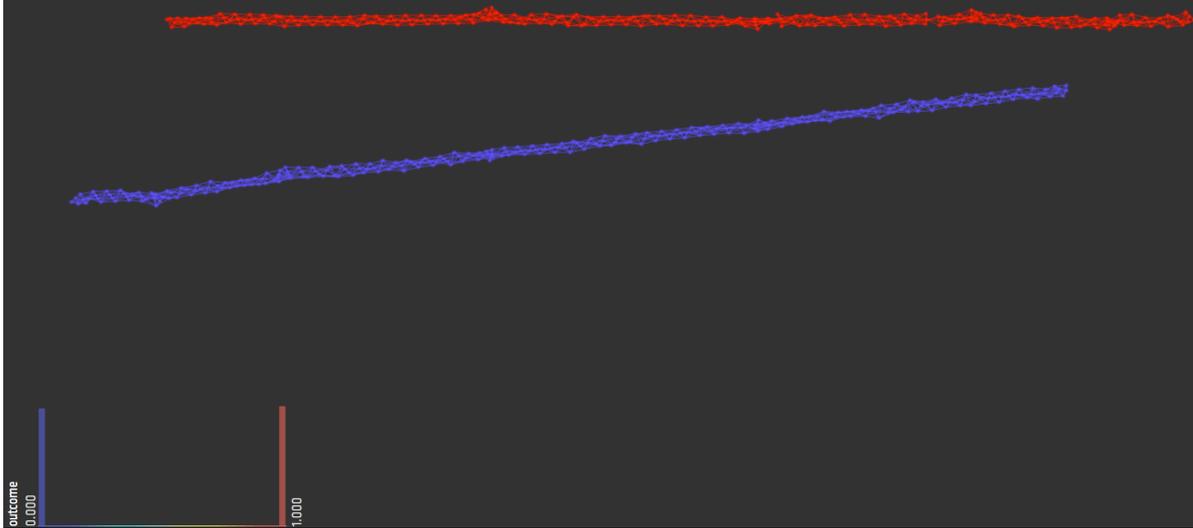


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- ▶ Separating the two classes was easy. Take connected components of graph.
- ▶ We retained more information than clustering. We remember that we have lines.
- ▶ If there was localized structure along the spiral, for example, subclasses of the two major classes, we would find those localizations on these lines.

### 3) Compressed Representation

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- ▶ Replace the metric space with a combinatorial summary: a simplicial complex.

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This is more or less what TDA is about

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  - ▶ Different problems require different summaries.

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- ▶ Measure what you find. Use as few assumptions as possible.
- ▶ Produce a summary relevant to the problem.
  - ▶ Different problems require different summaries.

Use your map to make decisions! Don't go back and measure from scratch.

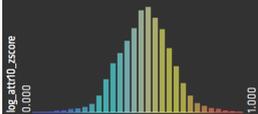
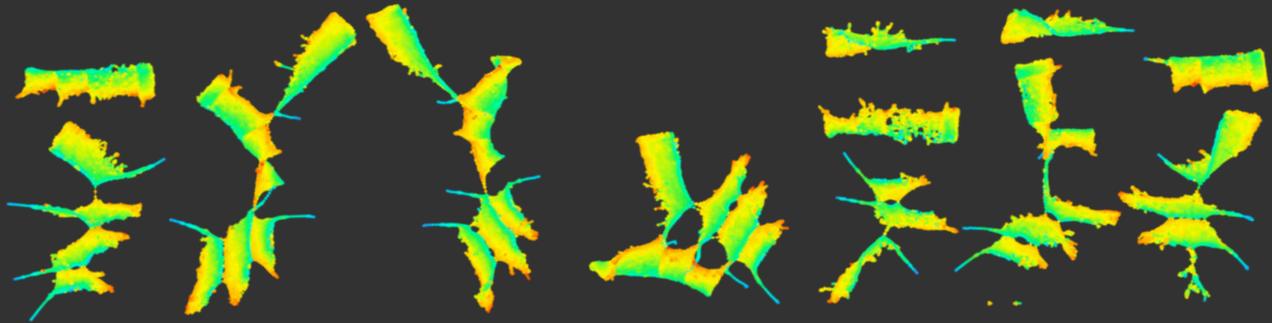
⇒ TDA is the machine that takes the tools (metrics & lenses) and produces the summary (network)

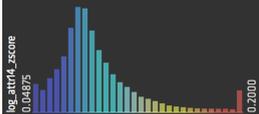
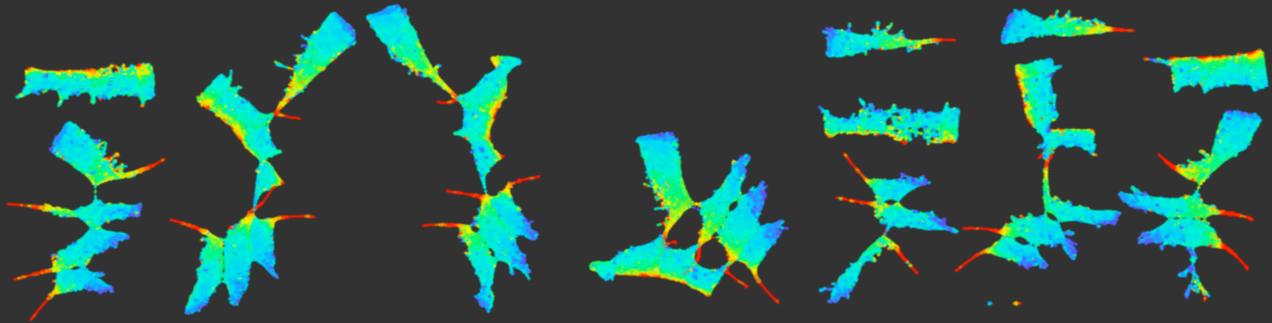
More Examples

## Customer Churn

**Data:** Customer usage and contractual details for major telco.

**Analysis:** A contractual stage data lens was used to split the data into "contractual stage" groups

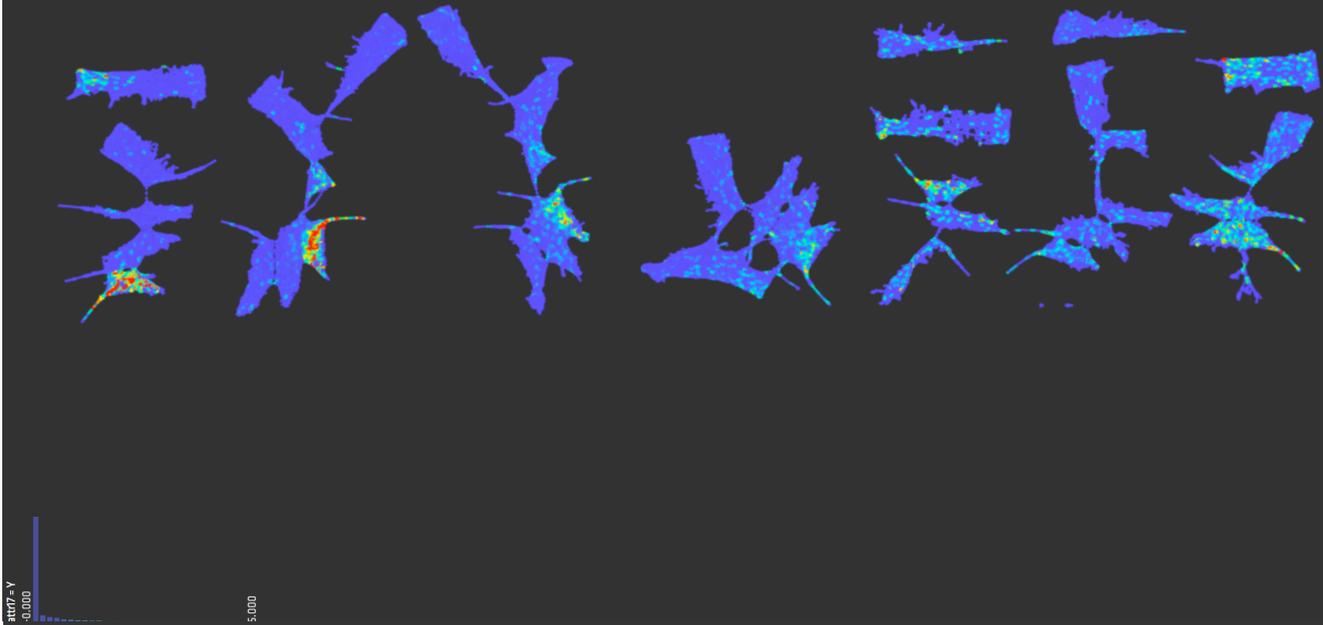




# Customer Churn

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# Customer Churn

Shape and Meaning

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- ▶ We see similar shape across all the contract stages.
- ▶ We can see natural segmentations that vary over many orders of magnitude (100-50,000 customers) in size.
- ▶ Stability gives us confidence in the validity of the results.

# Customer Churn

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⇒ We turn our insight into better targeting resulting in fewer lost customers. This can be automated.

## Predictive Maintenance: Industrial Machinery

**Setup:** We have a large piece of industrial machinery, think turbine, jet engine, locomotive or robot. Built into the machine are sensors measuring physical quantities: pressure, temperature, rpms etc.

**Problem:** Unscheduled downtime is very expensive.

**Question:** Can we predict when a part will need to be repaired in the future so we can schedule the downtime appropriately?

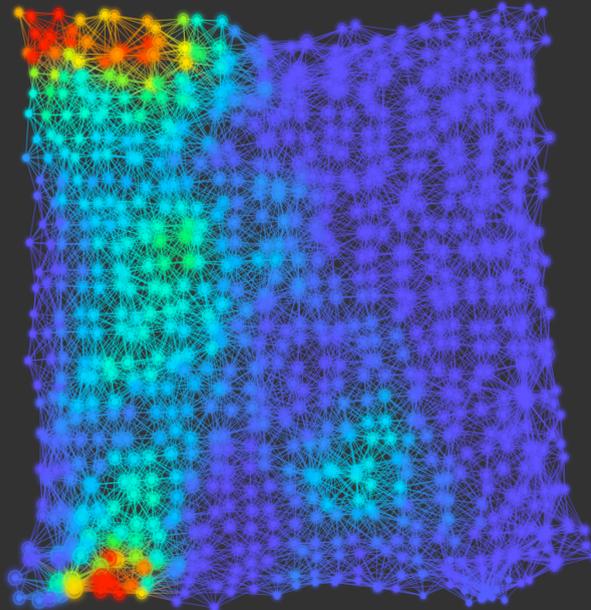
## Predictive Maintenance: Industrial Machinery

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## Predictive Maintenance: Industrial Machinery

**Data Transformation:** We want the sensors to be comparable. In this example, z-scoring is sensible (but there are other sensible choices as well, min/max normalization, logs if sensors vary of several orders of magnitude).

High mean, high variance



High mean, low variance

# Fraud Detection

AYASDI

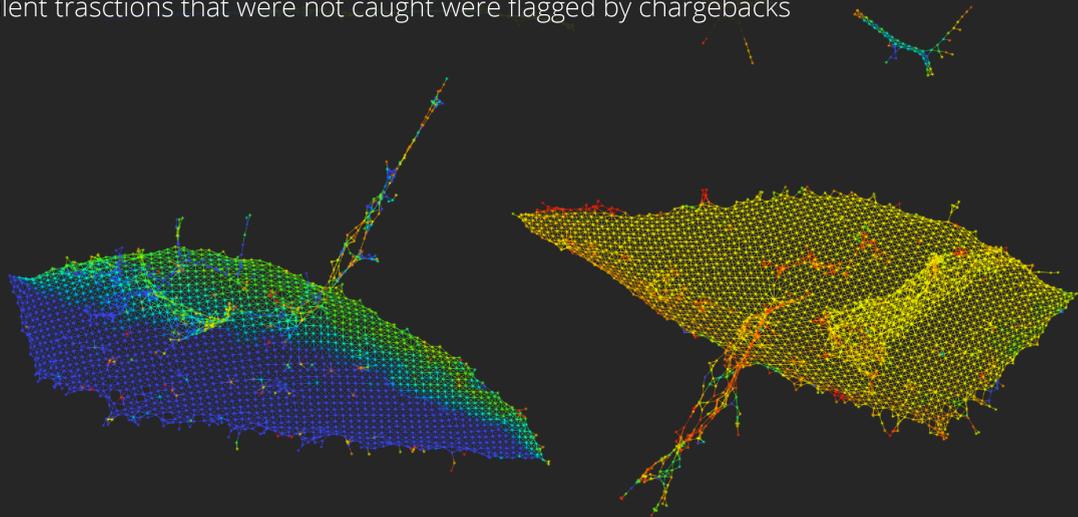
Risk score   
Low High

## About the data:

600,000+ transactions for a given month

Each transaction has 140 attributes (account, device, timing)

Fraudulent transactions that were not caught were flagged by chargebacks

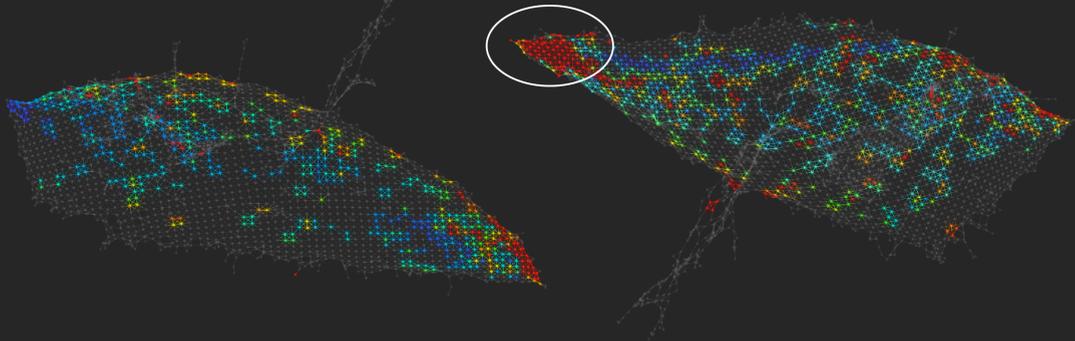


# Fraud Detection

AYASDI

Occurrence of Credit Charge Back   
Low High

Wherever the network lights up is  
a failure of the rules engine.



# Fraud Detection

# AYASDI

Occurrence of Credit Charge Back Low High

Categorical compare chargebac...\_S\_875\_BO\_C1.5' (130 column)

Filter

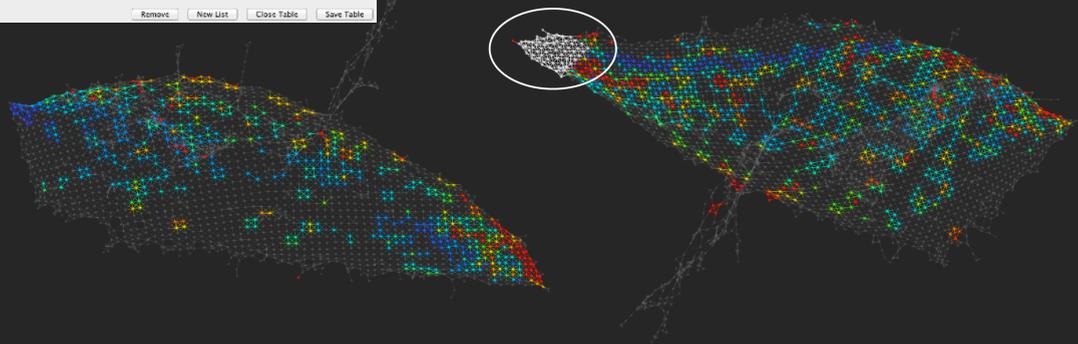
Column Name	Category Value	Selection Count	Background Count	p-value	IDs in selection
META_browser_string_hash	7791979e70b00	4177	8034	0E0	92,592,719,710
CAT_reason_code	1	9049	18249	0E0	72,59,290,405
CAT_reason_code	7	787	7013	0E0	70,92,224,36
CAT_reason_code	8	574	4413	0E0	70,90,92,224
CAT_reason_code	9	138	1854	0E0	70,92,269,52
CAT_reason_code	10	309	3455	0E0	260,1640,511
OUT_reason_code	7}Serveren Disa...	1184	2085	0E0	70,92,880,18
META_device_id_reason_code	IPGv0.1.5	3855	7127	0E0	70,92,224,36
CAT_risk_rating	low	5132	27233	0E0	70,92,213,36
headers_order_string_hash	898848218197	990	1391	0E0	224,398,587
CAT_device_match_result	not_enough_data	617	1116	1.74E-279	447,511,347
META_device_id_reason_code	IPGv0.1.5	1148	2668	6.74E-260	260,1680,111
META_device_id_reason_code	IPGv0.1.5	1107	2769	2.24E-216	260,1680,111
CAT_device_id	646	646	1376	6.58E-251	447,511,347
META_browser_string_hash	17285a9c3413	394	1043	1.88E-241	297,737,228
CAT_device_id	7	688	2412	1.23E-226	447,511,304
META_browser_string_hash	fd681d8e7c405	699	1672	4.27E-183	1076,2413,2
CAT_device_match_result	not_enough_data	440	789	6.42E-182	447,511,347
OUT_reason_code	review	482	1082	2.67E-139	447,511,271
OUT_risk_rating	medium	482	1082	2.67E-139	447,511,271
CAT_device_string	MediaSrv.com	293	486	1.14E-129	92,1016,278
CAT_reason_code	7}Cookies Disabl...	262	412	1.17E-112	1472,15311,
CAT_image_hosted	1	310	720	1.39E-104	1018,1049,9
OUT_reason_code	7}Cookies Disabl...	270	455	1.40E-100	447,511,462

Title: 2 of 400181 rows, 1 of 6 columns selected  
compare chargebacks from unknown to unknown in "V0\_LAND+NUM\_COSINE\_INF+CAIRS\_875\_EQ\_C1.5' (130 column)

Remove New List Close Table Save Table

Enriched for the following attributes:

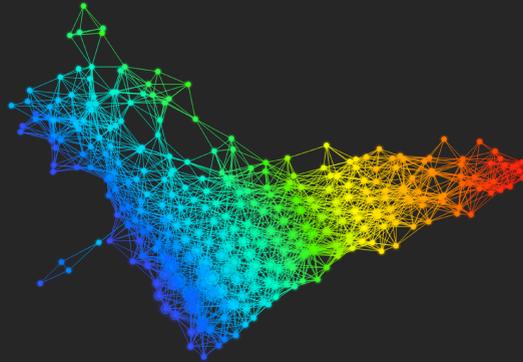
1. Images disabled
2. Javascript disabled
3. Cookies disabled
4. Flash disabled
5. Time spent on page was significantly longer



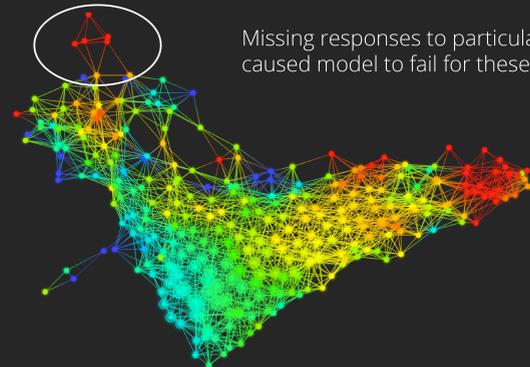
# Emergency room triage model

AYASDI

Predicted mortality  Low High

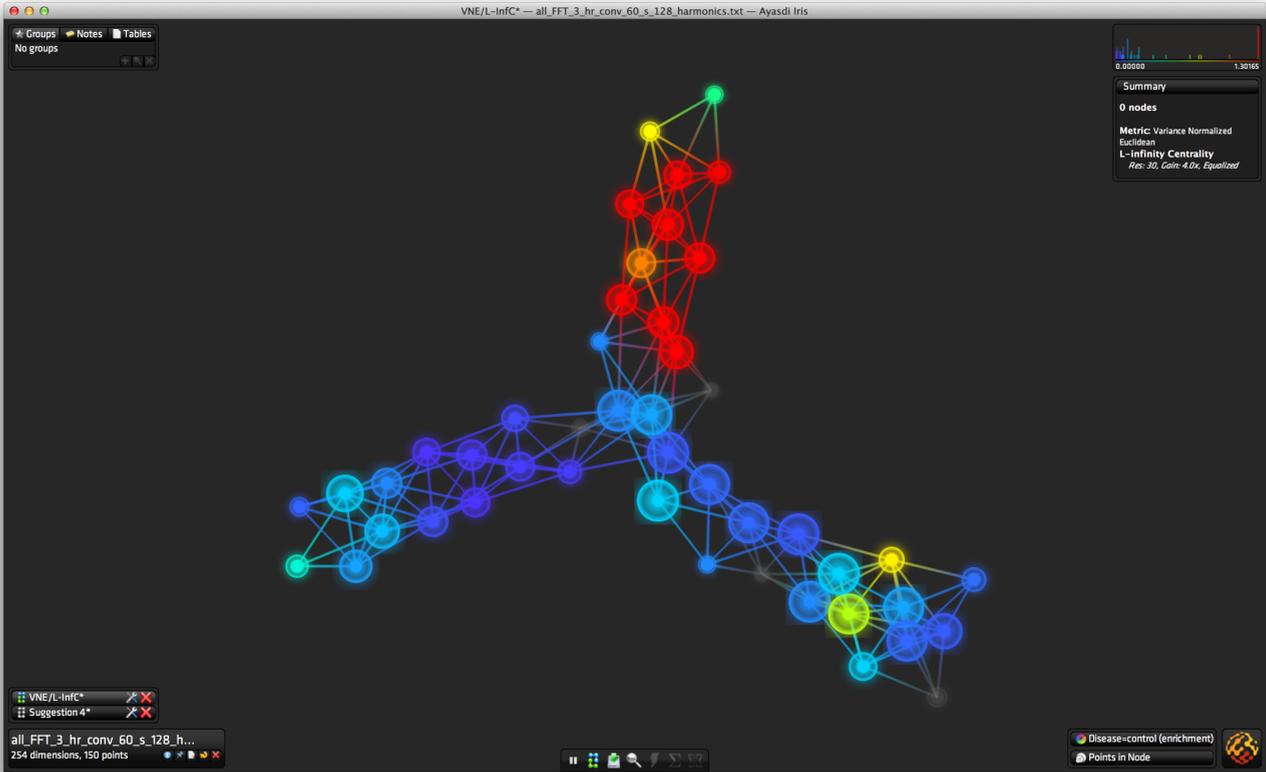


Actual mortality  Low High



Missing responses to particular questions caused model to fail for these patients

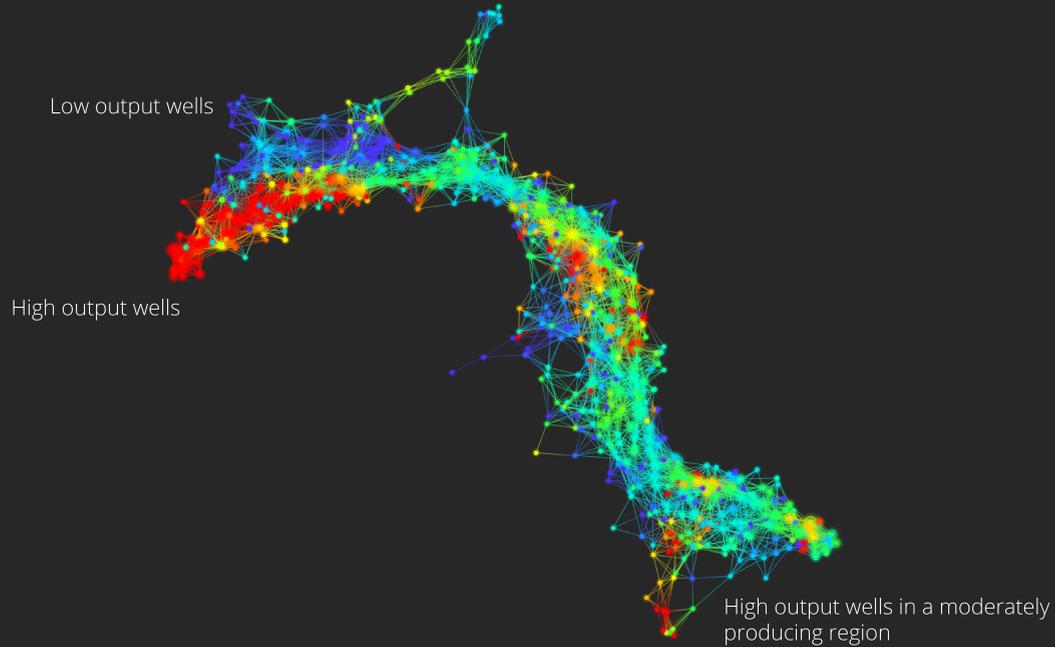
# Parkinson's Detection with Mobile Phone



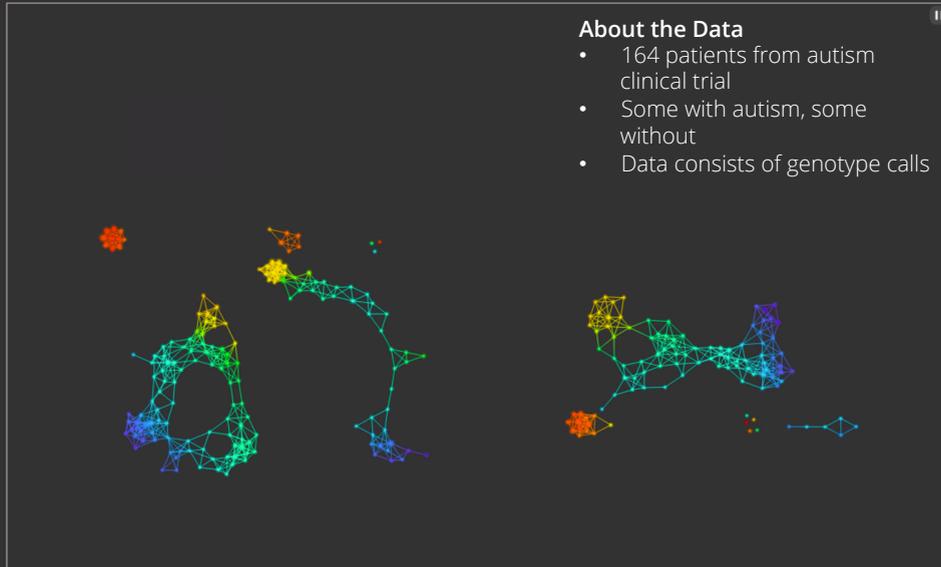
# Oil Well Sensors and Recovery

AYASDI

Well Production   
Low High

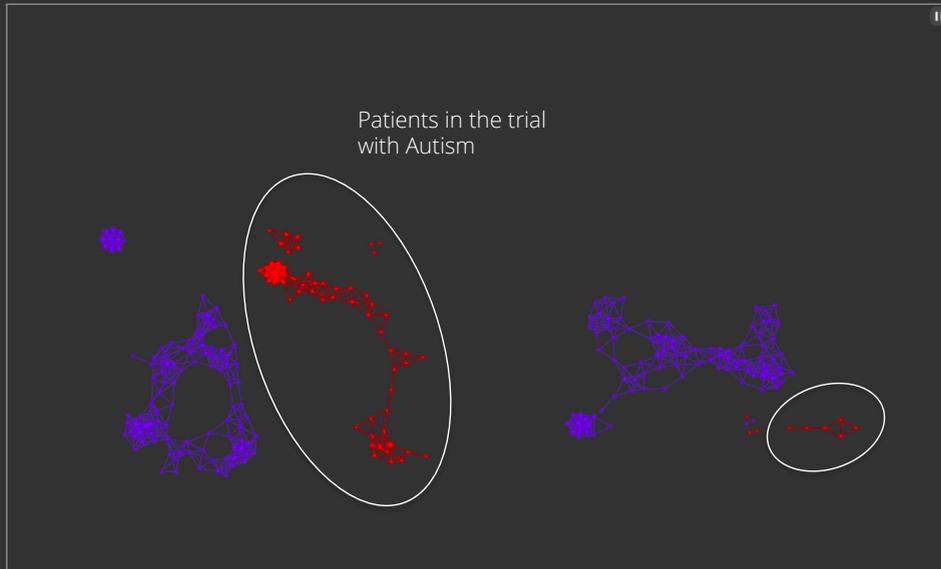


# Analyzing NGS Data with Ayasdi Cure



**Goal:** Identify genetic drivers of the disease in subpopulations

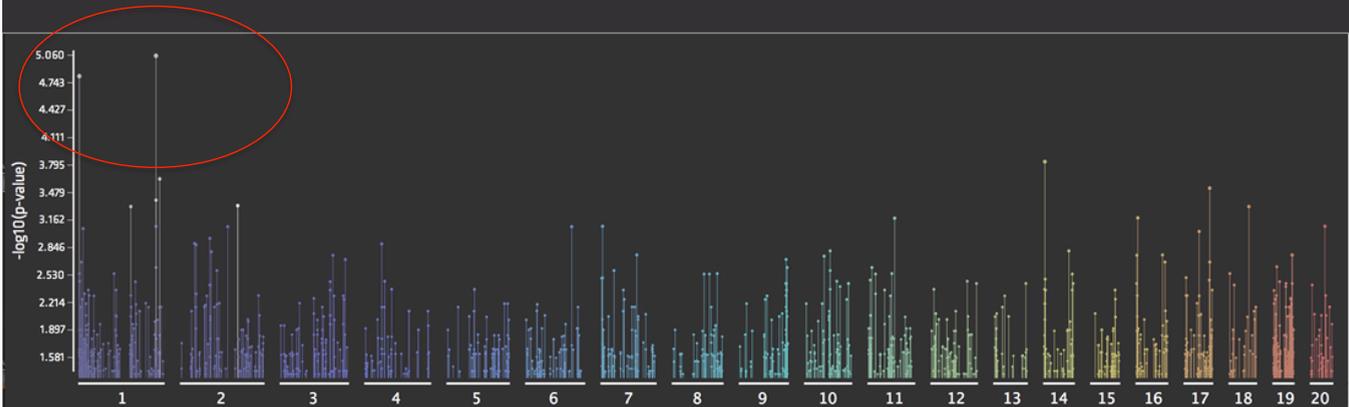
# Analyzing NGS Data with Ayasdi Cure



Disease Phenotype  
for Autism



# Analyzing NGS Data with Ayasdi Cure

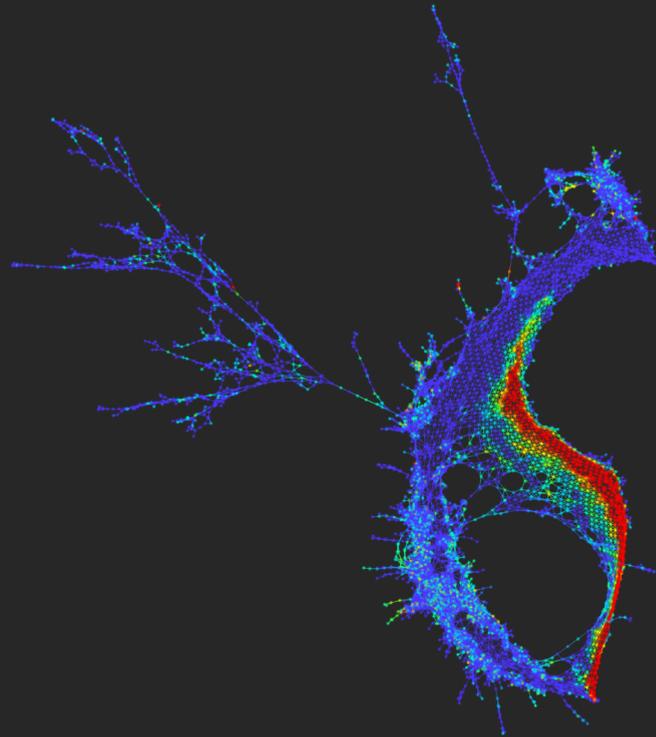


AYASDI

# The Wellcome Trust Case Control Consortium

**AYASDI**

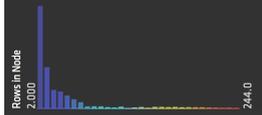
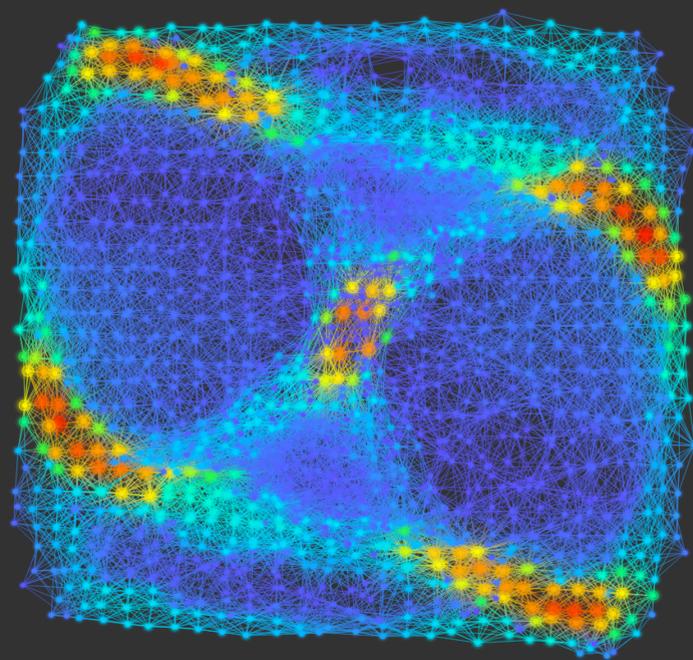
Variants with association to Crohn's Disease   
Low High



# Isomap: Configuration Space of $C_8H_{16}$

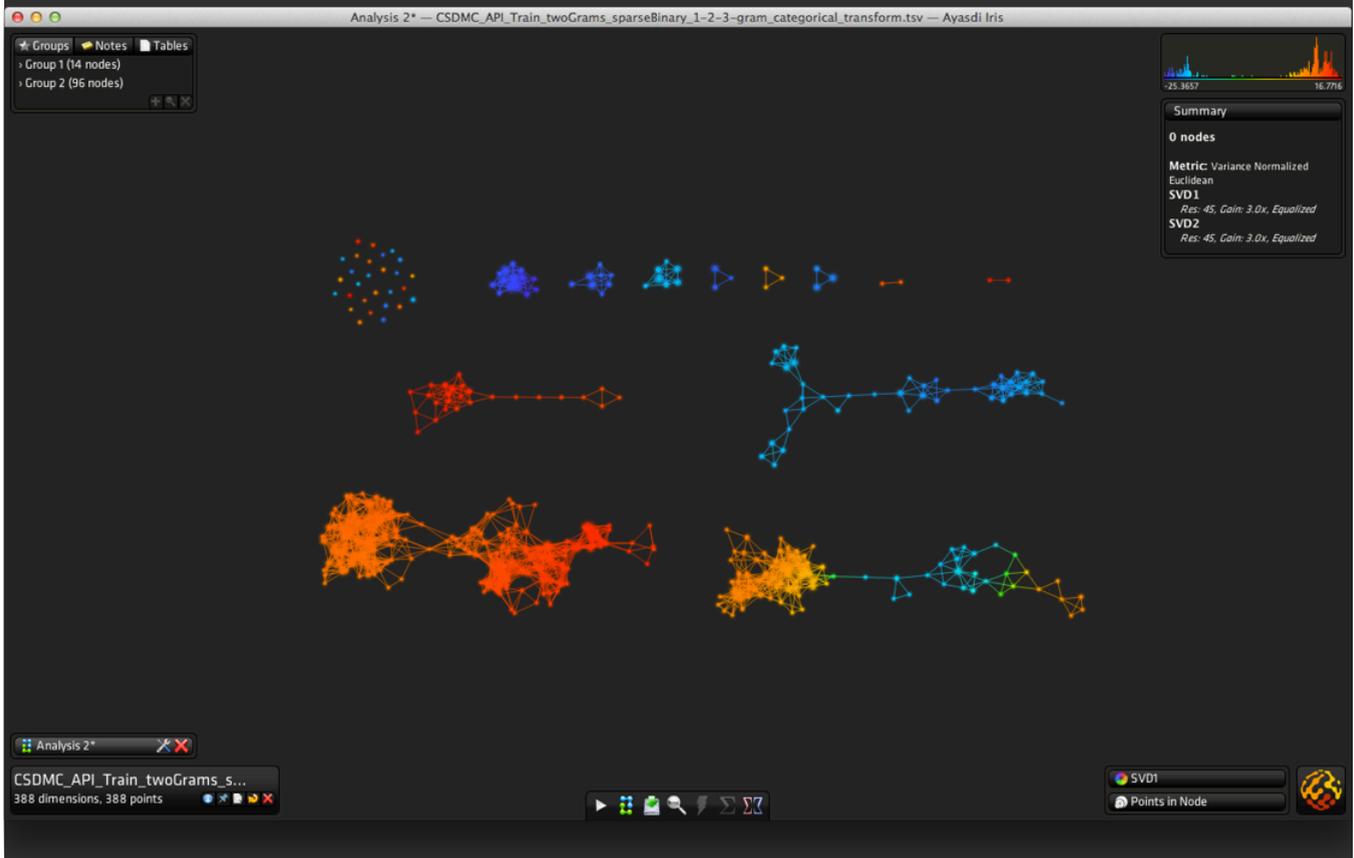
AYASDI

11



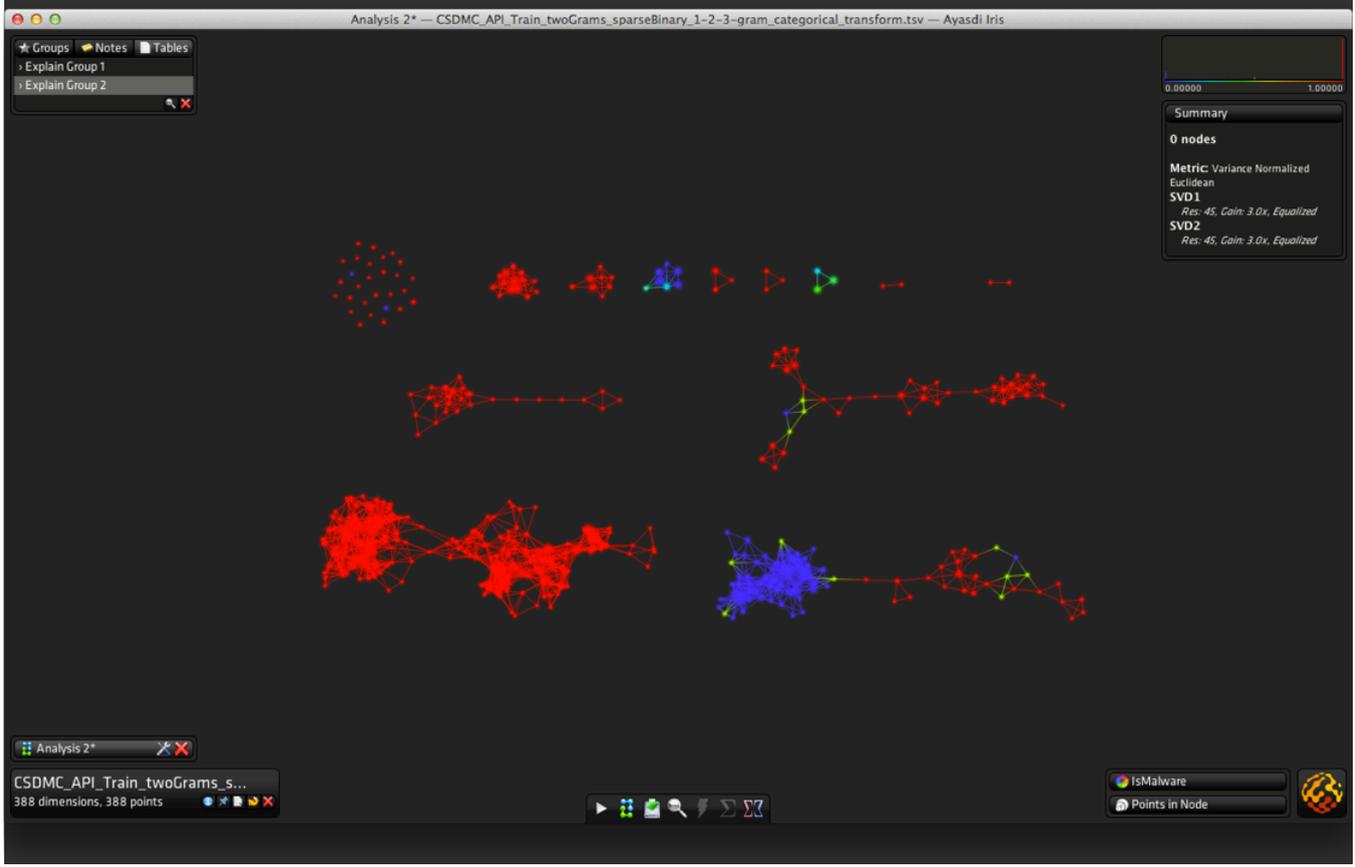
# Malware System Calls

# AYASDI

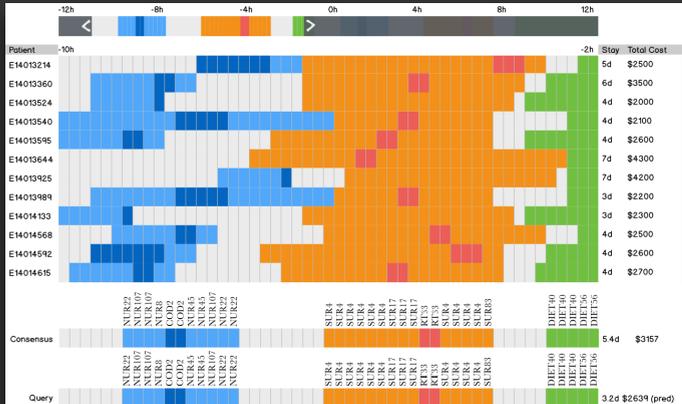


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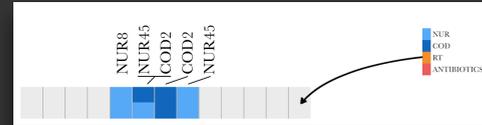
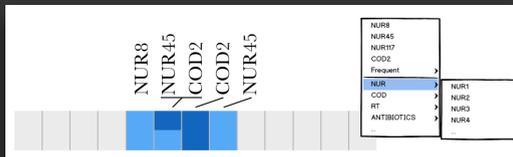


# User Experience for Care Paths



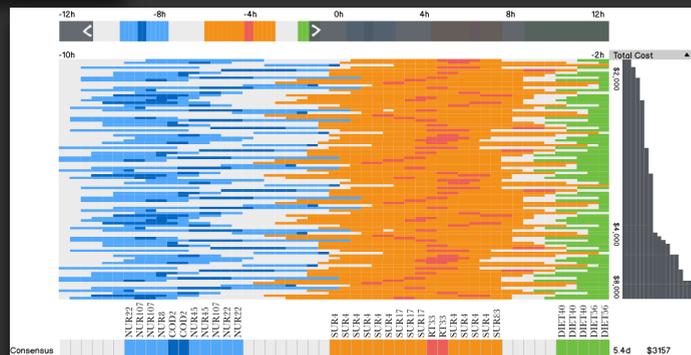
Patient Query

Patient Query Detail



Drag and Drop Interface

Care Path Overview



What's the point of all this?

Data Has Shape  
And Shape Has Meaning

Thank You!

<http://www.ayasdi.com/>